CMBS Default: A First Passage Time Approach

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Abstract

Empirical studies on CMBS default have focused on the probability of default depending on loan characteristics at the origination and market indices. Recent studies show current LTV is important in default prediction, although it is generally unobservable. This paper employs first passage time approach to study commercial mortgage backed securities default using implied LTV. It is defined as loan balance divided by one plus the return on the REIT index multiplied by property value, and follows geometric brownian motion process. Default is triggered as soon as the implied LTV crosses the exogenous threshold level.

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1 Introduction

Empirical studies\(^1\) on CMBS default have focused on probability of default depending on loan characteristics at the origination, and market indices. Once the explanatory variables are chosen, a logistics regression is run to find the parameter estimates, which are then used to calculate the default probabilities. Studies find LTV is positively correlated with default probabilities, and Current Loan-to-Value (CLTV) is especially important and dominates other variables in terms of its significance in default prediction.

As Capozza, Kazarian, and Thomson (1997) mentioned in their paper, there is not one way of defining CLTV. This is because current loan to value is not an observable variable. They define CLTV in terms of book value CLTV, and market value CLTV. Book value CLTV is the amortized loan balances divided by the current estimate of property value, and market value CLTV is defined by the the amortized loan balances divided by the value of property. One has to use market rate to discount the future payments to find the market value. One can also use price indices in the denominator. Haurin, Hendershott and Kim (1991) used house price series.

This paper employs first passage time approach to study commercial mortgage backed securities default using implied LTV. It is equal to the current loan amount divided by REIT indices implied value. Implied value at time \(t + 1\) is calculated by one plus the return on REIT indices from time \(t\) to time \(t + 1\), then multiplied by the value at time

\(^1\)Archer et.al. [2], Ciochetti et.al [6], Ciochetti et.al [7], Deng et.al [8], Vandell [15], Yıldırım [16]
ILTV follows geometric Brownian motion process. Default is triggered as soon as the implied LTV crosses the exogenous threshold level. The idea is modelling the evolution of the most significant variable in default risk and finding the default probabilities, rather than first finding the parameters of a logit model and then calculating default probabilities.

An outline of this paper is as follows: The next section presents the first passage time model. Section three describes the data and results. Finally, section four concludes the paper.

2 Modeling Default Probabilities

Estimating default probabilities are important for valuing risky securities. One can estimate them either directly using statistical methods, or modelling first, and then using an estimation method. Here, I will focus on modelling issues.

The credit risk literature has been essentially developed in two directions. In reduced form models, default occurs unexpectedly, and is explicitly defined by an intensity or compensator process.

In structural models, the default time is determined by an underlying process describing the firm value. This approach was introduced by Merton (1974) and has been extended by a number of authors. In his setup, the default event may occur only at the
debt’s maturity date. Specifically, at the maturity date $T$, if the total value of the firm’s assets is less than the notional value $L$ of the firm’s debt, the firm defaults and the debt holders receive the amount $V_T$. Otherwise, the firm does not default, and its liability is repaid in full. The first passage time approach extends the original Merton model by accounting that the default may occur not only at the debt’s maturity, but also prior to this date. In this setup, default occurs if the firm value crosses a (constant or random) barrier. These models were introduced by Black and Cox (1976). The asset level which triggers default can be imposed exogenously (Black and Cox (1976), Longstaff and Schwartz (1995)) or endogenously by having the shareholders optimally liquidate the firm (e.g. Leland (1994), Leland and Toft (1996), and Anderson and Sundaresan (1996)).

This section describes the structural model introduced by Black and Cox (1996). We define a new random variable called implied LTV, defined as loan balance divided by REIT indices implied value. It follows the geometric brownian motion process. One can use different kinds of processes, but geometric brownian motion ensures logarithmic return of this new variable is non-negative. Default is triggered as soon as the implied LTV crosses the exogenous threshold level. Figure (1) illustrates the first passage time model of geometric brownian motion in a simulated path. Value process, which follows geometric brownian motion, is initiated at $V_0$. Default happens the first time when this process hits the barrier of $b$. 
The relationship between default probabilities and REIT indices, and other loan specific variables are given in Yıldırım (2005). He shows while LTV is positively correlated with default, REIT indices are negatively correlated.

Let $\tau$ represents the time when a default occurs. It is defined as the first time the underlying process, $V$, crosses the barrier $b$

$$\tau = \inf\{t \geq 0 : V_t \geq b\}$$

(2.1)

where $b > 0$. Like in Merton (1974), $V$ follows a geometric brownian motion,

$$\frac{dV_t}{V_t} = \mu dt + \sigma dW_t$$

(2.2)

where $W$ is a standard brownian motion defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ initialized at $V_0 \leq b$. (2.2) can be rewritten as below

$$d(InV_t) = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW_t,$$

(2.3)
The default probability can be written as follows:

\[
P(\tau \leq t) = P\left( \max_{0 \leq s \leq t} V_s \geq b \right) \\
= \Phi \left( \frac{\ln \left( \frac{b}{V_0} \right) - (\mu - \sigma^2 / 2) t}{\sigma \sqrt{t}} \right) - e^{\frac{\mu - \sigma^2}{2}} \Phi \left( \frac{\ln \left( \frac{b}{V_0} \right) - (\mu - \sigma^2 / 2) t}{\sigma \sqrt{t}} \right),
\]

(2.4)

where \( \Phi(.) \) is the cumulative standard normal distribution. For the derivation, refer to Karatzas and Shreve (1991). To find the default probability of a loan, the drift, \( \mu \), and volatility, \( \sigma \), parameters of the equation (2.2) have to be calculated. Equation (2.3) implies the log return of the process is an independently drawn random variable from a normal distribution,

\[
\ln \left( \frac{V_t}{V_{t-1}} \right) \sim N \left[ (\mu - \sigma^2 / 2), \sigma^2 \right].
\]

(2.5)

Using the historical log returns of implied LTV, sample moments of \( \ln \left( \frac{V_t}{V_{t-1}} \right) \) can be calculated to find \( \mu \) and \( \sigma \) loan by loan.

\( V_t \) is chosen as ILTV. Under the above framework, one can chose a different variable and calculate the default probabilities using equation (2.4), after calculating the drift and volatility terms.
3 Estimation

3.1 Data

The commercial loan data set is provided by WOTN. WOTN data comes from independent and unrelated resources. Interest rates are from the Federal Reserve Board’s web site. REIT general stock price, property value, and property type by region index are from the National Council of Real Estate Investment Fiduciaries (NCREIF) and Bloomberg. CMBS loan prices, deal structure and loan characteristics are from Trepp’s comprehensive historical commercial loan database.

Trepp’s data has been provided to the rating agencies directly. WOTN gets all the loan level data that has been provided to the rating agencies by Trepp going back to the beginning of 1998, plus additional information going back to 1996. For the purposes of this study, the reliability and cleanliness of the data includes loans with origination dates beginning in 1995 but the time series that I use begins in 1998. Thus I do not incorporate the history from 1995 to 1998 because it is unreliable. Within this data set, I only use LTV and REIT general stock price index, and REIT property by region specific indices to find the new variable I called ILTV.

Table (1) summarizes the defaulted loan numbers in every property type by region, whereas Table (2) shows the loan distribution in every property type by region. I used

\(^2\text{http://www.wotn.us}\)
52,927 non-CTL (e.g. credit tenant lease) with fixed rate CMBS loans from June 1998 to May 2005 with monthly observations. There are 1,192 defaulted loans in the sample, which is 2.25% of the entire population. A high proportion of censored observations is observed. If one wants to use a statistical technique to calculate the default rates for such a heavy censored data, he has to be careful in applying the logit model. Yıldırım (2005) estimates the parameters in case of heavy censoring using the mixture model.

Although the loans are dense in the Pacific region, less default is observed. It has the second lowest percentage of total defaulted loan after Other type. Similarly, 31.7% loans are in the Multifamily property type, yet only 1.48% of these loans are defaulted. It has the second lowest percentage of total defaulted loan after Industry. In terms of region, West North Central has the highest defaulted loan rate with 4.41% and low loan numbers, and Other has the lowest defaulted loan rate with low loan numbers.

### 3.2 Results

First, the implied LTV values are averaged at the default date to find a proxy for the exogenous barrier. It is used to calculate the probability of default for each loan in equation (2.4). ILTV is calculated using REIT general stock price indices and REIT property by region stock price indices. Therefore, we have one fixed barrier for the first case, and 56 different barriers for the second case. We have six property types and nine regions, which adds up 56 barriers.
Then, the drift and volatility parameters of the process loan by loan are calculated using different REIT indices. Average drift and volatility terms using REIT general stock price indices are $-0.0082$ with a standard error of $0.0015$, and $0.0575$ with a standard error of $0.0080$ respectively. Average drift and volatility terms using REIT property by region stock price indices are $-0.0024$ with a standard error of $0.0025$, and $0.0277$ with a standard error of $0.0319$ respectively.

Figure (2) shows the estimated default probabilities of loans averaged region by region. Within each property type, estimated average default probabilities are 2.80%, 2.74%, 3.04%, 3.04%, 2.28%, and 2.97% for Industry, Lodging, Office, Other, and Retail respectively. With an average default probability of 3.04%, Multifamily has the highest value, and Other has the lowest value. This shows the higher number of loans within a property type has higher default probability. This is what is expected since the REIT general stock price indices are used, and they distribute the weight across property types equally.

In Figure (3), ILTV is calculated using REIT property by region specific indices. Within each property type, estimated average default probabilities are 0.25%, 2.85%, 0.59%, 1.23%, 4.12%, and 2.37% for Industry, Lodging, Office, Other, and Retail respectively. With an average default probability of 2.85% and 4.12%, Lodging and Other have the highest estimated default rates. This is what is seen in terms of the actual default rate ordering in the data. With 10.27% and 2.76% of default rates. With an average
default probability of 0.25%, Industry has the lowest estimated default rate. This is also similar to the actual default rate of Industry. Table (3) summarize these results.

4 Conclusion

This paper employs the first passage time model to calculate the default probabilities of CMBS loans. The geometric brownian motion process is used to ensure log returns of the implied LTV are non negative, and a closed form solution is derived. The evolution of the most significant term in default risk, e.g. implied LTV parameter, is modelled. Rather than using a logit model, we calculated default probabilities. It would be interesting to investigate more complex structural models than those used herein.
References


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### Table 1: Defaulted loan numbers property by region

<table>
<thead>
<tr>
<th>Region</th>
<th>IN</th>
<th>LO</th>
<th>MF</th>
<th>OF</th>
<th>OT</th>
<th>RT</th>
<th>% of total default</th>
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</thead>
<tbody>
<tr>
<td>East North Central</td>
<td>15</td>
<td>27</td>
<td>26</td>
<td>28</td>
<td>4</td>
<td>51</td>
<td>2.95</td>
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<tr>
<td>Mideast</td>
<td>2</td>
<td>61</td>
<td>33</td>
<td>8</td>
<td>5</td>
<td>24</td>
<td>2.50</td>
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<tr>
<td>Mountain</td>
<td>8</td>
<td>31</td>
<td>18</td>
<td>19</td>
<td>4</td>
<td>19</td>
<td>2.15</td>
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<tr>
<td>Northeast</td>
<td>13</td>
<td>18</td>
<td>15</td>
<td>24</td>
<td>11</td>
<td>46</td>
<td>1.46</td>
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<tr>
<td>Pacific</td>
<td>7</td>
<td>16</td>
<td>3</td>
<td>18</td>
<td>7</td>
<td>24</td>
<td>0.65</td>
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<td>Southeast</td>
<td>12</td>
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<td>46</td>
<td>14</td>
<td>14</td>
<td>71</td>
<td>3.24</td>
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<tr>
<td>Southwest</td>
<td>9</td>
<td>54</td>
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<td>20</td>
<td>4</td>
<td>82</td>
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<td>1</td>
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<td>6</td>
<td>8</td>
<td>33</td>
<td>20</td>
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<tr>
<td>Other</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.50</td>
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<tr>
<td>% of total default</td>
<td>1.15</td>
<td>10.27</td>
<td>1.48</td>
<td>1.82</td>
<td>2.76</td>
<td>2.04</td>
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### Table 2: Loan distribution property by region

<table>
<thead>
<tr>
<th>Region</th>
<th>IN</th>
<th>LO</th>
<th>MF</th>
<th>OF</th>
<th>OT</th>
<th>RT</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>East North Central</td>
<td>401</td>
<td>285</td>
<td>1721</td>
<td>591</td>
<td>212</td>
<td>1901</td>
<td>5111</td>
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<tr>
<td>Mideast</td>
<td>465</td>
<td>425</td>
<td>1163</td>
<td>862</td>
<td>347</td>
<td>2062</td>
<td>5324</td>
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<tr>
<td>Mountain</td>
<td>568</td>
<td>236</td>
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<td>699</td>
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<td>1545</td>
<td>4600</td>
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<tr>
<td>Northeast</td>
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<td>3124</td>
<td>1538</td>
<td>296</td>
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<tr>
<td>Southeast</td>
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<td>2323</td>
<td>718</td>
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<td>2677</td>
<td>7233</td>
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<td>Southwest</td>
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<td>2861</td>
<td>574</td>
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<td>2044</td>
<td>6609</td>
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<tr>
<td>West North Central</td>
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<td>742</td>
<td>233</td>
<td>302</td>
<td>574</td>
<td>2109</td>
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<tr>
<td>Other</td>
<td>230</td>
<td>189</td>
<td>286</td>
<td>152</td>
<td>580</td>
<td>377</td>
<td>2109</td>
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<tr>
<td>Total</td>
<td>5840</td>
<td>3076</td>
<td>16814</td>
<td>7678</td>
<td>2966</td>
<td>16553</td>
<td>52927</td>
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### Table 3: Average Default Rates: Actual, and Estimated using different REIT index

<table>
<thead>
<tr>
<th>Industry</th>
<th>Actual</th>
<th>Estimated Using REIT General</th>
<th>Estimated Using REIT Property by Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
<td>1.15%</td>
<td>2.80%</td>
<td>0.25%</td>
</tr>
<tr>
<td>Logging</td>
<td>10.27%</td>
<td>2.74%</td>
<td>2.85%</td>
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<tr>
<td>Multifamily</td>
<td>1.48%</td>
<td>3.04%</td>
<td>0.59%</td>
</tr>
<tr>
<td>Office</td>
<td>1.82%</td>
<td>3.04%</td>
<td>1.23%</td>
</tr>
<tr>
<td>Other</td>
<td>2.76%</td>
<td>2.28%</td>
<td>4.12%</td>
</tr>
<tr>
<td>Retail</td>
<td>2.04%</td>
<td>2.97%</td>
<td>2.37%</td>
</tr>
</tbody>
</table>
Figure 1: Simulated Geometric Brownian Motion initialized at $V_0$, with barrier $b$. 

\[ \tau = \inf \{ t \geq 0 : V_t \geq b \} \]

\[ V_t = V_0 e^{(\mu - \frac{1}{2} \sigma^2)t + \sigma W_t} \]
Figure 2: Average probability of default for all regions. Implied LTV is calculated using REIT general stock price indices.
Figure 3: Average probability of default for all regions. Implied LTV is calculated using REIT property by region stock price indices.