Dynamics of City Growth: Random or Deterministic?
Evidence From China

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Abstract

This paper uses Chinese city size data from 1984-2002 to test three urban growth theories: the random growth theory, the endogenous growth theory, and the hybrid random-endogenous growth theory. The results from unit root and cointegration tests on pooled heterogeneous cities in the country do not support any of the three growth theories. However, we find that certain groups of cities with common group characteristics do grow parallel.

Keywords: City growth, Random growth, Endogenous growth, Zipf’s Law, Unit root, Cointegration

JEL Classifications: C22, R11, R12

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1 Introduction

There are two striking facts about city growth. First, cities keep growing in different countries, in terms of both city sizes (city population) and the number of cities. Second, the distributions of city sizes in different countries fit the power law (Pareto distribution) very well. Especially, in the upper tail of city size distribution, the power exponent is equal to (or very close to) 1, which is called Zipf’s Law or rank-size rule (city size is proportional to the inverse of its rank).\footnote{There is rich literature on city size distribution. Let $R$ and $P$ denote the rank and population size of a city, then the power law or Pareto distribution implies that the number of cities whose population exceed $P$ is proportional to $P^{-\beta}$. In terms of econometric specification: $\ln R = \alpha - \beta \ln P + \epsilon$. With $\beta = 1$, it is called Zipf’s Law (Zipf, 1949) or rank-size rule. However, to be specific, rank-size rule is only a good approximation of Zipf’s Law (Gabaix, 1999). Many studies found that the upper tail of city size distribution obeys Zipf’s Law quite well. For a comprehensive survey on city size distribution, see Gabaix and Ioannides (2004).} Correspondingly, three strands of theories have been developed to explain the two stylized facts. The random growth theory (Gabaix, 1999) assumed city growth as a random walk and demonstrated that in steady state city size distribution obeys Zipf’s Law. The urban endogenous growth theory (Black and Henderson, 1999; Eaton and Eckstein, 1997) used human capital externalities as the driving force to explain the persistent and deterministic growth of cities. The hybrid urban growth theory (Rossi-Hansberg and Wright, 2003) employed both human capital externalities and stochastic productivity shocks. Under some restrictive conditions, the hybrid model can generate both balanced endogenous growth and city size distribution close to Zipf’s Law. Unfortunately, empirically, we are still not clear if city growth is random or deterministic. If it is deterministic, does the growth...
converge (small cities grow faster), diverge (large cities grow faster), or parallel (all cities grow at the same speed)?

This paper tests which theory is supported by empirical evidence from Chinese city size time series data. China has incurred rapid urbanization since 1980s. The urbanization rate increased from 23.01% in 1984 to 39.09% in 2002. Almost in every year, there are new cities incoming. Figure 1 in the appendix shows the growth of urban population and the number of cities. The dramatic change of Chinese economic structure and policies may have had strong impacts on the evolution of city sizes and size distribution. Some cities strongly benefit from the huge agglomeration economies from nearby super-large cities or city-belt; some other cities, however, still suffer from locational disadvantages. Cities in special economic zones have been blessed by favorable government economic policies and grow very fast. Figure 2-a shows that Shenzhen (in the special economic zone) grew faster than Nanchong (in western China) and Linfen (in middle China) although these three cities had similar sizes in 1984. Cities in the same region but of different sizes may have different growth patterns. Figure 2-b reflects the growth of three large-sized, medium-sized, and small-sized cities in middle

Following economic growth theory (Barro and Sala-i-Martin, 2004), if small cities grow faster than large cities without conditional on any other characteristics of economies, it is referred to as absolute convergence, meaning that all cities will converge to the same long-run steady state size. If different cities converge to their own steady state sizes, it is called conditional convergence. If small cities grow faster than large cities after holding fixed some other variables, such as initial human capital stock, government policies, it is called convergence. If the dispersion of city sizes (say, the standard deviation of sizes of a group of cities) declines over time, it is referred to as convergence.
China. These special features of Chinese city growth have attracted a few studies. Song and Zhang (2002) used 1991-1998 data and discussed the change of the power exponent of Chinese city size distribution. Anderson and Ge (2005) tested the distribution of Chinese city size and the structural break point of city size evolution. However, we have not found any study focusing on the dynamics of Chinese city growth.

In this study, we are particularly interested in identifying the dynamic patterns of city growth in China and testing the predictions from the three urban growth theories. The results from unit root and cointegration tests on the sizes of all cities in the country do not support any of them. However, after conducting more tests by different groups of cities in terms of size, region, and policy regime, we find that some cities with certain common group characteristics do grow parallel.

The rest of the paper is organized as follows: section 2 reviews the three urban growth theories and their empirical implications and evidences; section 3 describes the data set. Section 4 tests the random growth theory by looking at the time variations of the power exponent of rank-size rule; section 5 further tests the random growth theory by a set of unit roots tests; section 6 tests the urban endogenous growth theory through cointegration test. Section 7 discusses related issues and further research agenda; section 8 concludes.

2 Three Urban Growth Theories

The urban random growth theory assumes that city sizes grow stochastically (to be specific, follow a geometric Brownian motion), if at least for a certain range of size, the cities follow
Gibrat’s Law (the growth processes have the common expected city growth rate and a
common standard deviation), then in the steady state, the distribution of city sizes in that
range will follow Zipf’s Law with a power exponent of 1 (Gabaix, 1999; Cordoba, 2001,
2003). There are three ways to test the random growth theory. First, do growth processes
of cities follow Gilbrat’s Law, or in general, follow random walk? Second, does city size
distribution follow Zipf’s Law? Third, do temporary random shocks have permanent effects
on city size evolution? The negative answer to any of the three questions would cast doubt
on the random growth hypothesis.

Davis and Weinstein (2001) found that one of the most powerful shocks to city sizes
evolution in the history, the Allied bombing of Japanese cities during World War II, only
has temporary effects: most cities returned to their relative positions in the distribution of
city sizes within about fifteen years from the devastating destruction. This strongly strikes
at the random growth theory.

The urban endogenous growth theory predicts that cities grow parallel, meaning that
cities of different sizes grow at the same constant speed in steady state. The Black-Henderson
model (Black and Henderson, 1999) assumed localized information spillovers and human
capital accumulation as the engines of urban growth and produced endogenous sizes and
number of cities over time. Sizes of different types of cities grow at the same rate which
is proportional to the growth rate of human capital accumulation; the number of cities
of each type also grows at the same rate which equals the difference between the rate of
national population growth and the rate of city population growth. Eaton and Eckstein
(1997) constructed a similar model and also predicted that the growth of a system of cities is parallel, maintaining a constant relative size distribution of cities.

Eaton and Eckstein (1997) used France and Japan city size data and estimated the Markov transition matrix of city size evolution. They concluded that cities grow parallel with quite stable distribution which is close to the rank-size rule. However, there are a few problems in their study. First, they only used the top 40 urban areas. As many studies pointed out, the threshold of city size matters in estimating the power exponent. Second, as pointed out by Sharma (2003), they did not provide statistical inference concerning the estimated transition probabilities. It is very hard to know how large the diagonal transition probability should be to justify persistent and parallel growth. Third, they did not discuss the stationarity of the city size time series data. If sizes of individual city are not stationary, then the better way to test the co-movement of city growth would be to conduct cointegration test. Finally, the period of rapid industrialization and urbanization in each country also is accompanied with important economic structure and policy changes, which may have had persistent effects on later urban growth and different impacts on different cities. Therefore, structural changes should also be taken into account.

Other possibilities of urban growth would be convergence and divergence. During the process of urbanization, new small cities keep forming and catch up with large ones, so the size distribution of cities would become more even over time, or may converge to a common steady state size. In contrast, urbanization could take the form of the expansion of existing large cities, which implies that size distribution would be more unequal or diverge. Black
and Henderson (1997) modelled transitions as a stationary first-order Markov process and found that the relative size distribution of cities is astonishingly stable over time, with the actual distribution fluctuating little between decades and exhibiting no tendency to collapse (converge to a common city size), spread, go bimodal, and so forth. However, under some special constraints, in their another model (Black and Henderson, 1999) cities could achieve steady state levels, meaning that cities will grow and converge to a common stationary size. It is interesting that an early study by Rosen and Resnick (1980) suggested that large cities grow faster than small cities in most of the countries in their sample.

The hybrid model by Rossi-Hansberg and Wright (2003) combines both Black-Henderson model and Gibrat’s Law and is able to predict both city growth facts. A random total factor productivity shock is introduced to the model so that the balanced growth of city sizes is also random. Under two sets of restrictive conditions (eliminating physical capital or AK type model without human capital), the expected long run growth rate and variance are independent of city size, which fits Gibrat’s law. Under certain range of parameter values, the hybrid model can also generate distributions which deviate from Zipf’s Law, as uncovered by empirical literature. The hybrid theory was tested by Sharma (2003). Sharma used Indian decennial population census data from 1901-1991 and conducted unit root and cointegration tests. The conclusion is that city growth may be parallel in the long run, but the short-run growth may deviate from the long-term rate of growth due to exogenous shocks, and temporary shocks may take less than a decade to dissipate.\footnote{It is interesting to note that Sharma’s empirical study is done before Rossi-Hansberg and Wright’s theoretical work. However, a careful reader can find that Sharma’s paper actually provides some empirical}
Another related but more mixed and hard-to-test theory—locational fundamentals theory (Krugman, 1996; Fujita and Mori, 1996), is also worth noting. Locational characteristics may be considered randomly distributed over space (a spatial random process). They are the initial conditions which play a crucial role in shaping the formation and evolution of the size at that location. Even the initial conditions become unimportant any more, their effects may still persist, which is called the “path dependence effect” or the “lock-in effect” of some self-reinforcing agglomeration forces (Fujita and Mori, 1996). The strong location-specific advantage may even be able to revert the strong temporary shocks to city growth. However, this theory makes no clear prediction of the pattern of urban growth. Davis and Weinstein (2001) used the soon recovery of Japanese cities after World War II bombing as the confirmation of locational fundamental theory.

This paper focuses on identifying the growth pattern of Chinese cities. Do Chinese cities grow randomly, parallel, or with a constant long-run rate but short-run deviation? The results will provide empirical evidence to test the three urban growth theories.

3 Data

This study uses Chinese city size data from 1984 to 2002. The data are from each year’s Urban Statistic Yearbook of China and Urban Yearbook of China. City population is defined as the number of non-agricultural population in urban area of a city (by the permanent residence) at evidence for Rossi-Hansberg and Wright’s conclusion that cities grow parallel in steady state yet the growth processes follow Gibrat’s Law.
year-end. Non-agricultural population are those who engaged in non-agricultural vocations and brought up by non-agricultural staff. Some observations in 2001 used the definition of urban population for population Census in 2000 and incurred abnormal jumps in population levels. Those cities will not be considered when conducting econometric tests. 4

Chinese cities usually are classified into five size categories according to their population:

1. Super Large-sized Cities (With a population above 2,000,000 persons);
2. Extra Large-sized Cities (With a population between 1,000,000-2,000,000 persons);
3. Large-sized Cities (With a population between 500,000-1,000,000 persons);
4. Medium-sized City (With a population between 200,000-500,000 persons);
5. Small-sized Cities (With a population less than 200,000 persons).

By region, Chinese cities traditionally are assigned to one of the following three categories:

1. Eastern region, including 12 provinces and central municipalities: Beijing, Tianjin, Hebei, Liaoning, Shanghai, Jiangsu, Zhejiang, Fujian, Shandong, Guangdong, Guangxi, Hainan;
2. Middle region, including 9 provinces and autonomous regions: Shanxi, Neimenggu, Jilin, Heilongjiang, Anhui, Jiangxi, Henan, Hubei, Hunan;
3. Western region, including 10 provinces and autonomous regions: Sichuan, Chongqing, Guizhou, Yunnan, Xizang, Shanxi, Gansu, Ningxia, Qinghai, Xinjiang.5

4The national urban population is the sum of population of all cities, which will be affected by those jumps. Therefore, we replace the recorded country population in 2001 with a smoothed number.

5There are two provinces with the same English name "Shanxi". But in fact, their Chinese names are different. Here we use Shanxi and Shanxi to indicate that they belong to different regions.
During the transition to the market-oriented economy and opening to the world, Chinese government favored a certain number cities to implement reform and open policies. These include 16 open coastal cities and 4 cities in special economic zones (starting from 1980).

To study the effects of size, region, and policy regime, the following 3 sub-groups of cities are constructed: 6

Group 1: Cities in special economic zones and their counterparts—cities of similar sizes but in other regions;

Group 2: Cities in the same region but of different sizes;

Group 3: Cities of the same size but in different regions.

Table 1 lists all the cities we choose from the 3 sub-groups.

4 Time Variations of Zipf’s Exponent

The urban random growth theory states that if city growth processes follow random walk and Gibrat’s Law, then the steady state distribution will obey Zipf’s Law. Therefore, if the Chinese city size distribution is not consistent with Zipf’s Law, we would cast doubt on the random growth theory. Though Anderson and Ge (2005) concluded that Chinese city sizes are better described by a log-normal distribution, the simple rank-size rule still fits Chinese city sizes nicely. We estimate the following model for each year to test whether the

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6We assign a city into a size group according to the population level in 1984. The choice of cities is randomized in each region and size category.
Table 1

Selected Cities in Sub-groups

<table>
<thead>
<tr>
<th>City</th>
<th>Region</th>
<th>Size</th>
<th>City</th>
<th>Region</th>
<th>Size</th>
<th>City</th>
<th>Region</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shantou</td>
<td>1</td>
<td>4</td>
<td>Suzhou</td>
<td>1</td>
<td>3</td>
<td>Suzhou</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Hegang</td>
<td>2</td>
<td>4</td>
<td>Zibo</td>
<td>1</td>
<td>3</td>
<td>Zibo</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Xining</td>
<td>3</td>
<td>4</td>
<td>Huainan</td>
<td>2</td>
<td>3</td>
<td>Chaozhou</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Shenzhen</td>
<td>1</td>
<td>5</td>
<td>Jilin</td>
<td>2</td>
<td>3</td>
<td>Yingkou</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Linfen</td>
<td>2</td>
<td>5</td>
<td>Guiyang</td>
<td>3</td>
<td>3</td>
<td>Binzhou</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Nanchong</td>
<td>3</td>
<td>5</td>
<td>Kunming</td>
<td>3</td>
<td>3</td>
<td>Zhangzhou</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Xiamen</td>
<td>1</td>
<td>4</td>
<td>Chaozhou</td>
<td>1</td>
<td>4</td>
<td>Huainan</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Pingxiang</td>
<td>2</td>
<td>4</td>
<td>Yingkou</td>
<td>1</td>
<td>4</td>
<td>Jilin</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Liupanshui</td>
<td>3</td>
<td>4</td>
<td>Changzhi</td>
<td>2</td>
<td>4</td>
<td>Changzhi</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Zhuhai</td>
<td>1</td>
<td>5</td>
<td>Huangshi</td>
<td>2</td>
<td>4</td>
<td>Huangshi</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Houma</td>
<td>2</td>
<td>5</td>
<td>Zunyi</td>
<td>3</td>
<td>4</td>
<td>Sannmenxia</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Jinchang</td>
<td>3</td>
<td>5</td>
<td>Leshan</td>
<td>3</td>
<td>4</td>
<td>Yiyang</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Binzhou</td>
<td>1</td>
<td>5</td>
<td>Guiyang</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Zhangzhou</td>
<td>1</td>
<td>5</td>
<td>Kunming</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sannmenxia</td>
<td>2</td>
<td>5</td>
<td>Zunyi</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Note: For regions, 1-Eastern, 2-Middle, 3-Western.
For sizes, 1-Super large, 2-Extra large, 3-Large, 4-Medium, 5-Small.

The power exponent is indeed close to 1 or not:

\[
\ln R = \alpha - \beta \ln P + \epsilon, \quad (1)
\]

where \( R \) and \( P \) are the rank and population size of a city. The power exponent \( \beta \) is also called Zipf’s exponent. Table 2 reports the estimated coefficients \( \beta \).
### Table 2

**Time Variations of Zipf’s Exponent**

<table>
<thead>
<tr>
<th>Year</th>
<th>All cities</th>
<th>Cities (pop &gt; 500,000)</th>
<th>Cities (pop &gt; 2,000,000)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Num. of cities</td>
<td>β</td>
<td>Num. of cities</td>
</tr>
<tr>
<td>1984</td>
<td>295</td>
<td>0.8910</td>
<td>50</td>
</tr>
<tr>
<td>1985</td>
<td>324</td>
<td>0.8557</td>
<td>52</td>
</tr>
<tr>
<td>1986</td>
<td>342</td>
<td>0.8570</td>
<td>54</td>
</tr>
<tr>
<td>1987</td>
<td>382</td>
<td>0.8838</td>
<td>55</td>
</tr>
<tr>
<td>1988</td>
<td>434</td>
<td>0.9266</td>
<td>58</td>
</tr>
<tr>
<td>1989</td>
<td>450</td>
<td>0.9324</td>
<td>58</td>
</tr>
<tr>
<td>1990</td>
<td>468</td>
<td>0.9023</td>
<td>60</td>
</tr>
<tr>
<td>1991</td>
<td>479</td>
<td>0.9231</td>
<td>61</td>
</tr>
<tr>
<td>1992</td>
<td>514</td>
<td>0.9504</td>
<td>62</td>
</tr>
<tr>
<td>1993</td>
<td>568</td>
<td>0.9796</td>
<td>69</td>
</tr>
<tr>
<td>1994</td>
<td>622</td>
<td>1.0071</td>
<td>73</td>
</tr>
<tr>
<td>1995</td>
<td>640</td>
<td>1.0226</td>
<td>76</td>
</tr>
<tr>
<td>1996</td>
<td>665</td>
<td>1.0376</td>
<td>78</td>
</tr>
<tr>
<td>1997</td>
<td>666</td>
<td>1.0319</td>
<td>81</td>
</tr>
<tr>
<td>1998</td>
<td>665</td>
<td>1.0473</td>
<td>86</td>
</tr>
<tr>
<td>1999</td>
<td>665</td>
<td>1.0752</td>
<td>86</td>
</tr>
<tr>
<td>2000</td>
<td>660</td>
<td>1.0228</td>
<td>92</td>
</tr>
<tr>
<td>2001*</td>
<td>655</td>
<td>0.8088</td>
<td>289</td>
</tr>
<tr>
<td>2002</td>
<td>653</td>
<td>0.9901</td>
<td>108</td>
</tr>
</tbody>
</table>

*: The definition of city size in 2001 changed for some cities, but was reverted in 2002.

All regressions have good fit; the $p$-values for hypothesis $H_0: \beta = 1$ are all zeroes.

In Table 2, coefficient $\beta$ in the first column is estimated from the full sample. The power exponent has been increasing from quite less than 1 in 1984 to quite close to 1 in 2002. This implies that the overall city size distribution becomes more even and close to what Zipf’s
Law predicts. Since many empirical studies confirm that the exponent is sensitive to the sample choice and is close to 1 for the upper tail of size distribution (Rosen and Resnick, 1980; Eeckhout, 2005), we also estimate the model by only selecting cities of size greater than certain large threshold. The third column and the fourth column report the results for cities of size greater than 500,000 and 2,000,000 respectively. However, estimated power exponents are significantly different from 1, to be specific, significantly greater than 1. And the values of $\beta$ increase when we move the cutoff to the upper tail. This implies that larger cities distribute more evenly than what Zipf’s Law predicts. One possible explanation is that the Chinese government has implemented the policy that restricted the migration into large cities and promoted the development of small cities. Another explanation is that the current size distribution may not be in the steady state. In fact, the overall size distribution of Chinese cities has been evolving. In 2002 the urbanization rate in China is 39.09%, while urbanization rate of developed countries is above 70%. China is still in the period of rapid urbanization and the distribution of city sizes will keep evolving. Historically, the power exponent shows U-shaped pattern in many countries (Parr, 1985). Cross-country studies also show that Zipf’s exponent is significantly different from 1 (Soo, 2005; Nitsch, 2005). Therefore, we tend to make no conclusion about the random and endogenous growth theory by only looking at the evolution of the power exponent.

\footnote{In 2000, the urbanization rate of U.S. is 77.2%; Canada, 77.1%; Japan, 78.8%; Britain, 89.5%; France; 75.6% (Source: www.stats.gov.cn).}
5 Non-stationarity in City Size and Stationarity in City Growth

The sizes of a city at different time periods are mostly likely correlated due to reasons such as the durability of urban infrastructures, housing, and fixed costs of production. This implies that a temporary random shock to city size may have significant impact on future city growth. The random growth theory indicates that the growth of city size is a random walk, meaning that a temporary shock will have permanent effect on city growth. If we can observe that all shocks only have temporary effects on city size like Davis and Weinstein (2002), we will confidently reject the random growth theory.

To test the persistence of random shocks to city size boils down to the test of the stationarity of city size, or the test of unit root. Let $\ln P_{it}$ be the logarithmic of the population of city $i$ at time $t$, then the simplest specification is to assume that city size is a first order autoregressive ($AR(1)$) process,

$$\ln P_{it} = \phi_i \ln P_{i,t-1} + \varepsilon_{it},$$

where $\phi_i$ is the first-order autocorrelation coefficient and $\varepsilon_{it}$ is the random shock at time $t$. The augmented Dickey-Fuller (ADF) test for non-stationarity of population levels takes the form

$$\Delta \ln P_{it} = \gamma_i \ln P_{i,t-1} + \varepsilon_{it},$$
where $\gamma_i = \phi_i - 1$. The null hypothesis of non-stationarity $H_0 : \phi_i = 1$ (or $\gamma_i = 0$) is used against the alternative hypothesis $H_1 : \phi_i < 1$ (or $\gamma_i < 0$). If $\phi_i < 1$, in steady state, the logarithm of city population will converge to a constant.

Since the conclusion of unit root test is sensitive to the specification of model, we choose the optimal lags using Akaike information criterion (AIC). The specification we use is

$$
\Delta \ln P_{it} = \alpha_i + \beta_i t + \gamma_i \ln P_{i,t-1} + \sum_{j}^{k_i} \rho_{ij} \Delta \ln P_{i,t-j} + \varepsilon_{it}.
$$

(2)

Constant $\alpha_i$ and linear trend $\beta_i t$ control for the upward trending. $k_i$ is the number of lagged difference term for city $i$.

We first conduct ADF test for each individual city. Cities which used different population definition in 2001 are not included. Among 149 cities remaining in the sample since 1984, the population levels of 138 cities have unit roots at the 10% level. This remarkably shows that most cities sizes are not stationary. So is the national urban population. Table 3 reports the unit root test for national urban population and the selected cities.

It is well known that the power of unit root test based on single equation is poor, especially when the time series is short. Panel unit root tests with large $N$ can improve the power. The time length of our data set is not long. But the number of cities is large enough. Therefore, Im-Pesaran-Shine’s (2003) panel method is used to improve the power of unit root test. The cities which reject individual unit root test are not included in the panel. The value of Im-Pesaran-Shine’s test statistic is -1.619. Comparing it with critical values -2.28 ($\alpha = 0.1$), -2.32 ($\alpha = 0.05$), and -2.38 ($\alpha = 0.05$), we can not reject the $H_0$: all cities in the panel have
### Table 3

#### Unit Root Test for Logarithmic Population Levels of Country and Selected Cities

<table>
<thead>
<tr>
<th>City</th>
<th>ADF test statistic</th>
<th>City</th>
<th>ADF test statistic</th>
<th>City</th>
<th>ADF test statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>country</td>
<td>-0.643</td>
<td>beijing</td>
<td>0.191</td>
<td>sanmenxia</td>
<td>-1.792</td>
</tr>
<tr>
<td>hegang</td>
<td>0.669</td>
<td>binzhou</td>
<td>-1.572</td>
<td>shanghai</td>
<td>-1.745</td>
</tr>
<tr>
<td>houma</td>
<td>-2.898</td>
<td>changzhi</td>
<td>-0.705</td>
<td>shenyang</td>
<td>-2.931</td>
</tr>
<tr>
<td>jinchang</td>
<td>-2.29</td>
<td>chaozhou</td>
<td>-1.839</td>
<td>suzhou</td>
<td>0.871</td>
</tr>
<tr>
<td>linfen</td>
<td>-0.417</td>
<td>guangzhou</td>
<td>0.611</td>
<td>tianjin</td>
<td>-2.239</td>
</tr>
<tr>
<td>liupanshi</td>
<td>-2.269</td>
<td>guiyang</td>
<td>-2.393</td>
<td>wuhan</td>
<td>-1.608</td>
</tr>
<tr>
<td>nanchang</td>
<td>-2.328</td>
<td>hangzhou</td>
<td>-1.602</td>
<td>yingkou</td>
<td>-1.91</td>
</tr>
<tr>
<td>pingxiang</td>
<td>-1.373</td>
<td>luainan</td>
<td>-1.753</td>
<td>yiyang</td>
<td>-1.761</td>
</tr>
<tr>
<td>shantou</td>
<td>-1.314</td>
<td>huangshi</td>
<td>-1.028</td>
<td>zhangzhou</td>
<td>0.929</td>
</tr>
<tr>
<td>shenzhen</td>
<td>-0.637</td>
<td>jilin</td>
<td>-2.543</td>
<td>zibo</td>
<td>-1.387</td>
</tr>
<tr>
<td>xiamen</td>
<td>0.778</td>
<td>jinchang</td>
<td>-2.29</td>
<td>zunyi</td>
<td>-1.702</td>
</tr>
<tr>
<td>xining</td>
<td>-2.793</td>
<td>kunming</td>
<td>-1.3</td>
<td>daqing*</td>
<td>-3.55</td>
</tr>
<tr>
<td>zhuai</td>
<td>-1.633</td>
<td>lesan</td>
<td>-2.166</td>
<td>yinchuan*</td>
<td>-5.023</td>
</tr>
</tbody>
</table>

Note: The critical values of ADF test for our specification are -3.24, -3.6, -4.38 at the 10%, 5%, and 1% levels respectively, calculated by STATA. * denotes rejection at the 5% level.

We also conduct formal unit root tests for population growth rate using the first difference of logarithmic population level. The null hypothesis of unit root of growth rate is rejected for every city.

There are two implications from unit root tests. First, there exists no steady state size for most cities. This rejects the conditional convergence hypothesis; second, the city size is an $I(1)$ process and the rate of growth is an $I(0)$ process. Therefore we can’t reject the random growth theory as confidently as Davis and Weinstein (2002).
Even city sizes evolve in a non-stationary way, they still could move together as city growth is affected by many common factors, such as national economic growth or other national-wide macroeconomic factors. A special case is that cities grow parallel, which is tested in the next section.

6 Parallel Growth of Cities

The urban endogenous growth theory predicts that cities of different sizes grow parallel at the same speed. This implies that their population levels also move together with the national urban population. Since there exist short-run shocks to city growth, empirically, the pure parallel growth requires that for any time period the expected growth rate of all cities be the same.

Suppose $\ln P_{it}$ is a general AR(1) process with a unit root, drift, and time trend,

$$\ln P_{it} = \alpha_i + \beta_i t + \ln P_{i,t-1} + \varepsilon_{it},$$

then the expected growth rate at time $t$ is

$$E(g_{it}) = E(\ln P_{it} - \ln P_{i,t-1}) = \alpha_i + \beta_i t.$$  

By the same token, the expected growth rate of another city $j$ at time $t$ is

$$E(g_{jt}) = E(\ln P_{jt} - \ln P_{j,t-1}) = \alpha_j + \beta_j t.$$  

The pure parallel growth requires

$$\forall t, \alpha_i + \beta_i t \equiv \alpha_j + \beta_j t.$$
Therefore, parallel growth implies the same time trend across all cities. The equilibrium relationship between sizes of two cities with parallel growth is

$$\ln P_{it} = \alpha_i + \ln P_{jt}.$$ 

Since both $\ln P_{it}$ and $\ln P_{jt}$ are likely unit root processes, the cointegration relationship should be tested before making any estimation. The regression model for cointegration test is specified as

$$\ln P_{it} = \alpha_i + \gamma \ln P_{jt} + \varepsilon_{it},$$

(3)

where $P_{jt}$ is the population level of chosen reference city. If $\ln P_{it}$ and $\ln P_{jt}$ are cointegrated and $\gamma \neq 1$, then city $i$ grows at a different rate from the reference city $j$. If $\ln P_{it}$ and $\ln P_{jt}$ are cointegrated and $\gamma = 1$, this favors parallel growth.\(^8\)

We consider the following four groups, using Johansen-Juselius cointegration rank test.

(1) Choosing the national urban population as the reference, we test the parallel growth of the national population and all cities of all sizes and in all regions. The populations of all 138 individual cities having unit root cointegrate with the national population. Three cities, Mianyang, Taizhou, and Yueyang, have population growth parallel to national population growth. They are located in different regions but all belong to small or medium size categories. Panel (a) of Table 4 provides the estimated value of $\gamma$ and test statistic values

\(^8\)Sharma (2003) specified the following model to test parallel growth: $\ln P_{it} = \alpha_i + \delta t + \beta_i \ln P_{jt} + \varepsilon_{it}$. He concluded that $\beta_i = 1$ implies parallel growth. This is true only if the time trend $\delta$ is zero or $\delta$ is very small and can be neglected.
for cointegration and parallel tests of these three cities. Figure 3-a visualizes their parallel growth.

(2) Choosing Shanghai as the reference city, we test if six super large-sized cities grow parallel. This is to investigate the growth of cities of comparable sizes. The populations of four super large-sized cities cointegrate with Shanghai population. And Beijing displays growth parallel to Shanghai. This makes sense because Beijing and Shanghai are political and economic centers in north and south China. Panel (b) of Table 4 and Figure 3-b provide the test statistics and graphs.

(3) Choosing Kunming as the reference city, we test six cities in the western region. This is to investigate the growth of cities of different sizes but in the same region. The sizes of three cities cointegrate with Kunming population. Guiyang displays parallel growth with Kunming. Both of them are provincial capitals. See Panel (c) of Table 4 and Figure 3-c for test statistics and visualization.

(4) Choosing Shenzhen as the reference city, we test four cities in the special economic zones. Populations of other cities in the special economic zones cointegrate with Shenzhen population. From Panel (d) of Table 4 and Figure 3-d, Zhuhai has seemingly parallel growth with Shenzhen.

In summary, the findings here do not support parallel growth as predicted by the urban endogenous growth theory and the hybrid growth theory. However, we do find that a few cities with similar size, location, and policy regime show parallel growth.
Table 4

Parallel Growth Test for Logarithmic Population Levels by Group

a: Parallel growth—nation and cities population (Reference: country)

<table>
<thead>
<tr>
<th>City</th>
<th>Size</th>
<th>Region</th>
<th>Rank: 0</th>
<th>Rank: 1</th>
<th>$\gamma$</th>
<th>p-value ($H_0: \gamma = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mianyang</td>
<td>22.07</td>
<td>3</td>
<td>64.8233*</td>
<td>1.1017</td>
<td>0.9610</td>
<td>0.3704</td>
</tr>
<tr>
<td>taizhou</td>
<td>13.53</td>
<td>1</td>
<td>37.9510*</td>
<td>0.0062</td>
<td>0.9578</td>
<td>0.7491</td>
</tr>
<tr>
<td>Yueyang</td>
<td>22.83</td>
<td>2</td>
<td>68.8901*</td>
<td>0.3617</td>
<td>1.0242</td>
<td>0.4632</td>
</tr>
</tbody>
</table>

b: Parallel growth—cities of similar sizes (Reference: Shanghai)

<table>
<thead>
<tr>
<th>City</th>
<th>Size</th>
<th>Region</th>
<th>Rank: 0</th>
<th>Rank: 1</th>
<th>$\gamma$</th>
<th>p-value ($H_0: \gamma = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beijing</td>
<td>498.3</td>
<td>1</td>
<td>37.7231*</td>
<td>0.2847</td>
<td>1.0228</td>
<td>0.5729</td>
</tr>
<tr>
<td>Guangzhou</td>
<td>248.61</td>
<td>1</td>
<td>24.4450*</td>
<td>1.1142</td>
<td>1.2550*</td>
<td>0.0191</td>
</tr>
<tr>
<td>Shenyang</td>
<td>317.32</td>
<td>1</td>
<td>44.5201*</td>
<td>0.0473</td>
<td>0.5214*</td>
<td>0</td>
</tr>
<tr>
<td>Tianjin</td>
<td>412.38</td>
<td>1</td>
<td>12.1476</td>
<td>0.4787</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Wuhan</td>
<td>289.9</td>
<td>1</td>
<td>23.1610*</td>
<td>0.0587</td>
<td>1.1712*</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

c: Parallel growth—cities in the same region (Reference: Kunming)

<table>
<thead>
<tr>
<th>City</th>
<th>Size</th>
<th>Region</th>
<th>Rank: 0</th>
<th>Rank: 1</th>
<th>$\gamma$</th>
<th>p-value ($H_0: \gamma = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guiyang</td>
<td>87.13</td>
<td>3</td>
<td>17.8771*</td>
<td>0.3335</td>
<td>0.9608</td>
<td>0.4114</td>
</tr>
<tr>
<td>Hanzhong</td>
<td>14.89</td>
<td>3</td>
<td>6.2579</td>
<td>0.1335</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Jinchang</td>
<td>6.93</td>
<td>3</td>
<td>49.4409*</td>
<td>0.4550</td>
<td>1.3602</td>
<td>0.0022</td>
</tr>
<tr>
<td>Leshan</td>
<td>29.55</td>
<td>3</td>
<td>31.9680*</td>
<td>0.0000</td>
<td>0.7089</td>
<td>0</td>
</tr>
<tr>
<td>Zunyi</td>
<td>23.37</td>
<td>3</td>
<td>14.2054</td>
<td>0.2518</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

d: Parallel growth—cities in special economic zones (Reference: Shenzhen)

<table>
<thead>
<tr>
<th>City</th>
<th>Size</th>
<th>Region</th>
<th>Rank: 0</th>
<th>Rank: 1</th>
<th>$\gamma$</th>
<th>p-value ($H_0: \gamma = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shantou</td>
<td>47.66</td>
<td>1</td>
<td>37.6685*</td>
<td>1.4774</td>
<td>0.3831*</td>
<td>0</td>
</tr>
<tr>
<td>Xiamen</td>
<td>32.81</td>
<td>1</td>
<td>26.6607*</td>
<td>2.2993</td>
<td>0.3990*</td>
<td>0</td>
</tr>
<tr>
<td>Zhuhai</td>
<td>6.77</td>
<td>1</td>
<td>63.5113*</td>
<td>2.8443</td>
<td>0.9631</td>
<td>0.3174</td>
</tr>
</tbody>
</table>

Note: The size is the population in 1984; Rank:0 and Rank:1 are cointegration ranks. The critical values are 15.67 and 9.24 respectively. *: significant at the 5% level.
7 Further Discussion

There are a few important issues worth further discussion.

First, the time series of each city is relatively short. Even for the panel unit root test, the $T$ of our sample is still small. One possible remedy is to expand the time span of the data set. But we are aware that before 1981 China employed different definition of urban population. This requires adjustment of either the data or the bias of coefficient estimates. Another possible solution is to look for more advanced panel data tools which do not require a large $T$, since we have a large number of units $N$. The factor model of panel unit root test proposed by Bai and Ng (2004) may be a good candidate.

Second, we only considered the balance part of our data set, i.e., cities having observations since 1984. However, one of the striking features of Chinese urbanization is the entry of new cities each year. It would be interesting to investigate the growth evolution incorporating the birth of new cities.

Third, observation of similar growth patterns in the three sub-groups may partly support the locational fundamental theory, since regional characteristics and policies in economic special zones are location-specific.

8 Conclusion

This paper focuses on identifying the growth patterns of Chinese cities and testing the three urban growth theories. Given the fact that China is still in the period of rapid urbanization,
even through we apply rigorous time series econometric techniques, we can only tentatively conclude that the overall Chinese city growth does not follow either random growth or parallel growth. However, a small number of cities with certain common group characteristics do grow parallel. For example, super large-sized cities Beijing and Shanghai grow parallel. Zhuhai and Shenzhen, both in special economic zones, also grow parallel. Although we are not clear what the city size distribution will look like when China’s urbanization becomes stable, our methodology should be able to produce more interesting and convincing empirical regularities when more data become available.
References


Figure 1-a: Growth of national urban population

Figure 1-b: Growth of city number
Figure 2-a: Growth of City in the Special Economic Zone

Figure 2-b: Grow of Cities with Different Sizes
Figure 3-a: Parallel growth of three cities populations and national population

Figure 3-b: Population growth of super large-sized cities
Figure 3-c: Population growth of western cities

Figure 3-d: Population growth of four cities in the special economic zones