Equilibrium Asset Price Correlation

(preliminary and incomplete)*

Charles Ka Yui Leung†

March 31, 2005

*Acknowledgement: Discussion with Min Hwang and Kelvin Wong motivates this research. The author is also grateful to the comments from Chor-yiu Sin and Miki Seko. The financial support from Chinese University of Hong Kong Direct Grant, RGC Earmark Grant, Fulbright Foundation are gratefully acknowledged. The usual disclaimer applies.

†Correspondence: Leung, Department of Economics, Chinese University of Hong Kong, Shatin, Hong Kong; (Phone) 852-2609-8194; (Fax) 852-2603-5805; (E-mail) charlesl@cuhk.edu.hk. (to be added)
Abstract

Two striking empirical regularities are repeatedly reported in the literature: (1) the price and return of stock, as well as housing, are significantly serially correlated, and (2) the return of housing and stock are correlated, and housing fails to provide a hedge for stock investment. Seminal work of Wheaton (1999) shows that when the housing stock adjusts sluggishly, the housing price and return will be serially correlated. This paper builds a tractable, unifying framework with forward-looking owner-occupiers-investors, and is capable to mimic the reported empirical regularities. Implications and future research directions are then discussed.

Keywords: rational expectation, price and return, serial and cross correlation, market efficiency

JEL Classification Number: E30, G10, R20
1 Introduction and Motivation

Two striking empirical regularities are repeatedly reported in the literature: (1) the price and return of stock, as well as housing, are significantly serially correlated, and (2) the return of housing and stock are correlated, and housing fails to provide a hedge for stock investment. The first regularity attracts a lot of attention especially after the work of Case and Shiller (1989, 1990). There is a large literature trying to understand whether, why and how the market is inefficient.\footnote{The literature is too large to be reviewed here. Among others, see Bollerslev and Hodrick (1995), Gatzlaff and Tirtiroglu (1995) for a survey.} The second regularity also leads to a large literature as researchers attempt to justify why residential housing possess such a large share in a typical household portfolio.\footnote{Again, that literature is too large to be reviewed here. Among others, see Hwang and Quigley (2003).} As observed by Hwang and Quigley (2003), these stylized facts are often quoted as evidence for market inefficiency, or even a basis to question the rationality of economic agents.

Not all economists agree on that interpretation though. For instance, Wheaton (1999) shows that when the housing stock adjusts sluggishly, the housing price and return will be serially correlated. He only considers the case of rental housing and hence it is not clear whether the agents are forward-
looking. This paper builds a tractable, unifying framework in which the owner-occupiers-investors are explicitly modeled to have rational expectation. The production of goods, and accumulation of physical and household capital are endogenized.\textsuperscript{3} We find that even with \textit{i.i.d.} shocks, our model can mimic the reported empirical regularities. In other words, the two empirical facts cannot be used as evidence against market efficiency. We will also discuss alternatives for testing market efficiency in some recent literature.

Since the focus of this paper is the dynamic behavior of prices and returns, it is appropriate to model them as endogenous rather than exogenous variables. In particular, we build a dynamic general equilibrium model with equity and housing. This choice is intentional, partly to self-impose more discipline in the modelling of asset-prices, and partly to reflect the fact that the asset prices and the fundamental are correlated, as suggested by some recent empirical research (especially over a longer horizon).\textsuperscript{4}

\textit{to be added}

\footnote{Throughout the paper, the terms "residential capital", "household capital", "housing" will be used interchangably.}

\footnote{For instance, see McQueen and Roley (1993), Wongbangpo and Sharma (2002), Parker and Julliard (2005).}
1.1 A Selective Literature Review

to be added

2 A Simple Model

Our model is built on Rebelo (1991), and Greenwood and Hercowitz (1991), Kan et. al. (2004), and hence the description will be brief.\textsuperscript{5} Time is discrete in this model and the horizon is infinite. The economy is populated by a continuum of infinite-lived agents. The population is fixed over time. In each period \( t, t = 1, 2, 3, \ldots \), the representative agent derives utility \( u(C_t, H_t + H_t^r, L_t) \) from non-durable consumption goods \( C_t \), the stock of housing (or residential property)\textsuperscript{6} owned (rented) by the agent \( H_t \) (\( H_t^r \)), as well as the amount of leisure enjoyed \( (1 - L_t) \), where \( L_t \) is the amount of time the agent supplied in the market, \( 0 \leq L_t \leq 1 \). \( H_t \) is broadly defined to include the residential structure, as well as the associated amenities. Following Greenwood

\textsuperscript{5}See also Kwong and Leung (2000), Leung (2001) for related studies.

\textsuperscript{6}In this paper, “housing” and “residential capital” will be used interchangeably.
and Hercowitz (1991), it is assumed that

\[ u(C_t, H_t, L_t) = \ln C_t + \omega_1 \ln (H_t + H^r_t) + \omega_2 \ln (1 - L_t), \]

where \( \omega_1, \omega_2 > 0 \). The representative agent is assumed to maximize the life time utility \( \sum_{t=0}^{\infty} \beta^t u(C_t, H_t + H^r_t, L_t) \), participate in the production of consumption goods \( C_t \), and accumulate business capital stock \( K_t \), and residential property \( H_t \). \( \beta \) is the time discount factor, \( 0 < \beta < 1 \).\(^7\)

The representative agent is a price taker in all industries and he/she is subject to a series of constraints. (To ease the notations, time subscripts are suppressed unless there is a risk of confusion). First, the total value of non-durable consumption \( C \), and investment in business capital, residential property, \( I_k, I_h \) respectively, the expenditure on the residential property purchased from the market, \( P_h H^m \), the expenditure on renting residential property, \( R_h H^r \), the expenditure on purchasing new equity, \( P_s (S_{t+1} - S_t) \), cannot exceed the total value of rental income from capital, \( R K \), labor income \( W L \), and dividend \( S \Pi^d \), where \( P_h \) is the relative price of housing (in terms of consumption goods), \( P_s \) is the relative price of equity, \( R \) is the factor

\(^7\)See Stokey, Lucas and Prescott (1989, esp. chapter 3) for more discussion on the role of the time discount factor.
return for capital, $W$ is the real wage rate, $\Pi^d$ is the total dividend.

Residential investment $I_h$ includes maintenance, renovation, purchase of new furniture, appliance, etc.\(^8\) Analogous interpretation applies to $I_k$ as well. Whether it pays to invest or where to invest depends crucially on the payoff of the investment. Following Hercowitz and Sampson (1991) and Benassy (1995), we assume a specific form of law of motion for different types of capital, which will generate closed forms of the solution,\(^9\)

\[ K_{t+1} = (K_t)^{1-\delta_k} (I_{ht})^{\delta_k}, \]  
\[ H_{t+1} = (H_t + H_{mt})^{1-\delta_h} (I_{ht})^{\delta_h}, \]  

where $0 < \delta_k, \delta_h < 1$. The dynamic programming problem of the representative agent is now:

\[ V(K_t, H_t, S_t) = \max_u u(C_t, H_t, L_t) + \beta V(K_{t+1}, H_{t+1}, S_{t+1}) \]  

\(^8\)Downing and Wallace (2002) show that the expenditures for home improvements total at least 2% of the GDP and vary systematically over the business cycle.\(^9\)An alternative approach is to adopt a more general framework and then use loglinearization as in Campbell (1994). The reduced forms of the dynamics, however, would be similar. See Lau (2002).
s.t. \( R_tK_t + W_tL_t + S_t\Pi^d_t \geq I_{kt} + I_{ht} + C_t + P_{ht}H^m_t + R_{ht}H^r_t + P_{st}(S_{t+1} - S_t) \), (5)

and (2), (3). It is implicitly assumed in (5) that the representative agent observes the current period productivity \( A_t \) first, and then decides how much raw materials are to be imported from the “rest of the world”, given the amount of capital and property the representative agent owns.

The production side of the economy is simple. Output are produced by combining capital and labor through a concave function,

\[
Y_t = A_t (K_t)^{\alpha_1} (L_t)^{\alpha_2} \tag{6}
\]

where \( A_t > 0, \forall t, 0 < \alpha_1, \alpha_2, \) and \( \alpha_1 + \alpha_2 < 1 \). The productivity is assumed to have finite mean and variance,

\[
0 < E(A_t), Var(A_t) < \infty.
\]

The factor markets are assumed to be competitive and the factor returns are equal to the marginal product,

\[
R_t = \frac{\partial Y_t}{\partial K_t}, W_t = \frac{\partial Y_t}{\partial L_t}. \tag{7}
\]
It is further assumed that the dividend is equal to the profit \( \Pi_d^t = \Pi_t \), which is the output net of factor payment,

\[
\Pi_t = Y_t - R_t K_t - W_t L_t. \tag{8}
\]

As it is standard in the growth model, the objective of the representative firm is to maximize the profit and hires capital and labor from the factor market accordingly.

To solve the model, it is necessary to impose market clearing conditions. Following Lucas (1978), the net trade of housing is assumed to be zero (both ownership market and rental market) and the total amount of equity is assumed to be unity in every period,

\[
S_{t+1} = S_t = 1, \ H_t^e = H_t^r = 0. \tag{9}
\]

In the appendix, we show that the problem can be simplified and prove the following proposition:

**Proposition 1** In this model economy, if

\[
\beta \alpha_1 \delta_k \left( 1 - \beta (1 - \delta_k) \right)^{-1} < 1, \tag{10}
\]
then the amount of working hours, the consumption and different kinds of investment shares of the output are constant,

$$L_t = L, \ C_t = S^cY_t, \ I_{j,t} = S^jY_t, \ j = k, \ h.$$ \hspace{1cm} (11)

The asset prices depend on the output, investment and housing stock,

$$P_{ht} = \left( \frac{1 - \delta_h}{\delta_h} \right) \left( \frac{I_{ht}}{H_t} \right),$$ \hspace{1cm} (12)

$$R_{ht} = \left( \frac{1 - \beta(1 - \delta_h)}{\beta(1 - \delta_h)} \right) P_{ht},$$ \hspace{1cm} (13)

$$P_{st} = \frac{\beta(1 - \alpha_1 - \alpha_2)}{1 - \beta} \cdot Y_t.$$ \hspace{1cm} (14)

With reasonable market-clearing conditions imposed, this proposition enables to characterize the equilibrium quantities (and prices) in each period as functions of exogenous variables. In other words, we can trace the evolution of the whole system.

It is convenient to rewrite in log form, i.e., we write $c_t = \ln C_t, \ y_t = \ln Y_t,$

$$p_{ht} = \ln P_{ht}, \ p_{st} = \ln P_{st}, \ s^j = \ln S^j, \ j = c, k, h,$$ etc. The economy is hence represented by the following linear equations:
\( y_t = \theta_y + \alpha_1 k_t + a_t, \) \hspace{1cm} (15)

\[
\begin{align*}
  k_{t+1} &= (1 - \delta_k) k_t + \delta_k i_{kt}, \\
  h_{t+1} &= (1 - \delta_h) h_t + \delta_h i_{ht},
\end{align*}
\] (16) (17)

\[
\begin{align*}
  c_t &= \eta_c + y_t, \\
  i_{kt} &= \eta_k + y_t, \\
  i_{ht} &= \eta_h + y_t,
\end{align*}
\] (18) (19) (20)

for some constants \( \theta_y \), and given the initial conditions \( a_0, k_0, h_0 \). And from (12), (14), we have the following corollary.

**Corollary 2** In log form, the stock price is linearly correlated to the output,

\( p_{st} = \theta_s + y_t \). \hspace{1cm} (21)

**Corollary 3** The (unconditional) serial correlation between different period
stock prices is positively correlated to the counterpart of the aggregate output,

\[ \text{cor} (p_{st}, p_{s,t+j}) = c_1 + c_2 \cdot \text{cor} (y_{st}, y_{s,t+j}), \quad j = 0, 1, 2, ... \quad (22) \]

where \( c_2 > 0 \).

By taking log of (12), we also have the following equation:

\[ p_{ht} = \theta_h + i_{ht} - h_t, \quad (23) \]

where \( \theta_s = \ln (1 - \alpha_1 - \alpha_2) \), \( \theta_h = \ln \left( \frac{1 - \delta_h}{\delta_h} \right) \). From (13), we also get

\[ r_{ht} = \theta_{rh} + p_{ht}. \quad (24) \]

Thus, to understand the dynamics of rent, it suffices to characterize the dynamics of housing price.

It is clear that the dynamical system is block-recursive and once we can dictate the joint dynamics of \( y_t \) and \( k_t \), we will also be able to pin down the dynamics of all other variables. In the appendix, the following results are proved.
Lemma 4 If \( 0 < E(a_t), Var(a_t) < \infty \), the joint dynamics of \( \vec{y}_{t+1}' = (y_t, k_t) \) can be described by the following vector equation,

\[
\vec{y}_{t+1} = \mathcal{M}_0 + \mathcal{M}_1 \vec{y}_t + \mathcal{M}_2 \vec{a}_{t+1},
\]
where \( \vec{a}_{t+1} = (a_{t+1}, 0) \), and for some constant matrices \( \mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2 \).

With (25), it is now possible to simulate the evolution of output, business capital, and commercial property \( (y_t, k_t)' \). In fact, we show in the appendix that the output, the capital stock, the housing stock, and the housing price, can all be written as a summation of previous period productivity shocks. Equipped with these results, we can now dictate the dynamics of the (log) housing price.

Lemma 5 The (log) housing price can be written as a function of the current and previous period output, or productivity shocks,

\[
p_{ht} = \theta_h + y_t + \delta_h \left( \sum_{i=0}^{\infty} (1 - \delta_h)^i y_{t-1-i} \right),
\]
\[
= \theta_h + \sum_{i=0}^{\infty} \delta_p(i) a_{t-i},
\]
for some function of parameter, \( \delta_p(i), i = 0, 1, 2, \ldots \).
Now, it is clear that the stochastic structure of the productivity shocks $\{a_t\}$ is crucial in determining the correlations among different variables. It would be no surprise if a serially correlated productivity shock leads to serially correlated prices. Therefore, for expositional purpose, we will mainly focus on the case with serially uncorrelated productivity shocks. We will show that even in that case, the asset prices will display serial correlation in a dynamic general equilibrium framework. In other words, there is an internal propagation mechanism behind the asset prices. Again, the appendix contains the proofs.

**Proposition 6** Even with serially uncorrelated productivity shocks, the serial correlation of asset prices will in general be non-zero,

$$
\text{cov}(p_{st}, p_{s,t+j}) = \sigma^2_a \left( \alpha_k (1 - \delta_k + \alpha_1 \delta_k)^{j-1} + \frac{(\alpha_1 \delta_k)^2 (1 - \delta_k + \alpha_1 \delta_k)^j}{1 - (1 - \delta_k + \alpha_1 \delta_k)^2} \right) > 0, \quad (28)
$$
where $\sigma_a^2 \equiv \text{var}(a_t)$ and for $j = 1, 2, 3,...$. And for housing prices,

$$
cov(p_{ht}, p_{h,t+j}) = \sigma_a^2 \left( \sum_{i=0}^{\infty} \delta_p(i) \cdot \delta_p(i+j) \right),
$$

(29)

for $j = 1, 2, 3,...$. Moreover, the two assets are correlated contemporarily,

$$
cov(p_{st}, p_{ht}) = \sigma_a^2 \cdot \left[ \sum_{i=0}^{\infty} \delta_p(i) \delta_p^s(i) \right],
$$

(30)

where $\delta_p^s(i) > 0, \forall i$, are functions of parameters. In general, the correlation between the current stock price and the subsequent period housing prices are non-zero. For $j = 1, 2, 3,...$

$$
cov(p_{st}, p_{h,t+j}) = \sigma_a^2 \left[ \sum_{i=0}^{\infty} \delta_p(j+i) \delta_p^s(i) \right],
$$

(31)

Conversely, the correlation between the current period stock price and previ-
ous period housing prices is

\[ \text{cov}(p_{s,t}, p_{h,t-j}) = \sigma_a^2 \left[ \sum_{i=0}^{\infty} \delta_p(i) \delta_p^*(j + i) \right]. \quad (32) \]

For economic and financial research, it is natural to study not only the dynamic behavior of prices, but also of the rate of returns. We define the rate of return of stock and housing as follows.

\[ R_{s,t+1} = \frac{P_{s,t+1} + \Pi_{t+1}^d}{P_{st}}, \quad R_{h,t+1} = \frac{P_{h,t+1} + R_{h,t+1}}{P_{st}}. \quad (33) \]

With these definitions, we can prove the following proposition.

**Proposition 7** In log form, the rate of return of stock can be written as a function of output,

\[ \tilde{r}_{s,t+1} = -\ln \beta + y_{t+1} - y_t, \quad (34) \]

and that of the housing can be written as a function of housing price,

\[ \tilde{r}_{h,t+1} = -\ln (\beta (1 - \delta_h)) + p_{h,t+1} - p_{ht}. \quad (35) \]
In addition, we can show that, for \( j = 1, 2, 3, \ldots \)

\[
\text{cov}(\tilde{r}_{s,t+1}, \tilde{r}_{s,t+1+j})
\]
\[
= \sigma_a^2 \cdot \left\{ \delta_p^s(0) \delta_r^s(j) + \sum_{i=0}^{\infty} \delta_r^s(i+1) \cdot \delta_r^s(j+i+1) \right\},
\]

(36)

\[
\text{cov}(\tilde{r}_{h,t+1}, \tilde{r}_{h,t+1+j})
\]
\[
= \sigma_a^2 \cdot \left\{ \delta_p^s(0) \delta_r^s(j) + \sum_{i=0}^{\infty} \delta_r^s(i+1) \cdot \delta_r^s(j+i+1) \right\},
\]

(37)

where \( \delta_r^s(i) \equiv [\delta_p^s(i) - \delta_p^s(i-1)] \), \( \delta_r^s(i) \equiv [\delta_p^s(i) - \delta_p^s(i-1)] \). In addition, we can examine the cross-correlation of the two assets’ return. In particular, the contemporaneous cross-correlation is given by the following formula,

\[
\text{cov}(\tilde{r}_{s,t+1}, \tilde{r}_{h,t+1})
\]
\[
= \sigma_a^2 \cdot \left\{ (\delta_p^s(0) \cdot \delta_p(0)) + \sum_{i=0}^{\infty} \delta_r^s(i+1) \cdot \delta_r(i+1) \right\}.
\]

(38)
And for $j = 1, 2, 3, \ldots$ the covariance between the current period stock return and the subsequent period housing return is

$$
cov(\tilde{r}_{s,t+1}, \tilde{r}_{h,t+1+j}) = \sigma_a^2 \cdot \left\{ (\delta_p^s(0) \cdot \delta_r^s(j)) + \sum_{i=0}^{\infty} \delta_r^s(i+1) \cdot \delta_r^s(i+j+1) \right\}, \quad (39)
$$

and the covariance between the current period stock return and the previous period housing return is

$$
cov(\tilde{r}_{s,t+1}, \tilde{r}_{h,t+1-j}) = \sigma_a^2 \cdot \left\{ (\delta_r^s(j) \cdot \delta_p^s(0)) + \sum_{i=0}^{\infty} \delta_r^s(j+i+1) \cdot \delta_r^s(i+1) \right\}, \quad (40)
$$

(to be added)

(to be added)

3 Concluding Remarks

The efficiency of asset markets, or markets in general, have long been questioned. For instance, Lamont and Thaler (2003) review some recent literature
on the violation of the “Law of One Price” in asset markets. Baye, Morgan and Scholten (2004) do not even find evidence for the “Law of One Price” on the internet, in which the information cost should be minimal. On the other hand, the research agenda of establishing the in-efficiency of the market by studying the asset price or asset return correlation may need to be refined. As shown in Wheaton (1999), sluggish adjustment in (rental) housing stock itself is enough to generate housing price auto-correlation. This paper generalizes this insights in several dimensions. We find that even when agents have rational expectation, the equilibrium prices and return of assets (housing and equity) will be correlated. Furthermore, the cross-correlation of the prices and return of the two assets will be non-zero. This research seems to encourage further effort to understand how asset prices and returns would be correlated in a general equilibrium setting.

The model developed in this paper may also be of independent interest. In fact, our model is so tractable that in the case of iid shock, we can provide closed form solutions for those correlation. We believe that it can be extended and modified in different directions for the investigation of other issues in financial economics.
References


Appendix:
(Note for the editors: In case it is decided that the appendix need not be published, it will be available upon request)

A Proofs

to be added