Revisit the Coase Conjecture:

Monopoly, Durability, and Bundling in Urban Land Use

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(ABSTRACT)

Although land and collective goods are bundled together, they could be provided separately. This paper studies intertemporal externality in land monopoly and analyzes the interaction between land market structure and the provision of local public goods. I consider four institutional settings depending on whether land and collective good are provided in a bundle: bundled rental, separate rental, bundled sale, and separate sale. By incorporating both intertemporal externality and “public good” externality, the two-period model developed in this paper can provide clues to many important land use phenomena such as the conflict between development interest and current residents. It also suggests that homeownership may result in more land development than leasehold. The quality of collective good provision in different institutional settings depends on cost vis-à-vis consumer valuation. Numeric examples demonstrate (1) separate provision, i.e., the common form of government providing collective goods, may be efficient for some range of parameterization such as more uniform distribution at large spatial scales; (2) rentals can be rather desirable for “poor” communities, such as downtowns and historic company towns; (3) bundled sale, such as CID (Common Interest Development) and condominium, is more efficient for “rich” communities. These results may help to explain why most private communities are small-scale and located in the suburbs.

KEYWORDS: monopoly, durability, bundling, land, local collective good, private community, urban institutions
INTRODUCTION

When Coase (1972) introduced his famous conjecture on the relationship between monopoly and durability, he used land as the example and assumed a monopolist who owns all land in America. However, in the following literature in industrial organization land has almost completely disappeared. Given the strong interest in the privatization of local public services, this paper revisits the Coase Conjecture from a land economic and institutional perspective. The goal is to formally analyze intertemporal behavior in land use and explore the relationship between market structure and urban institutions.

The Coase conjecture is about a monopolist who may price discriminate his customers over time. Since he does not internalize the impact of his behavior on the price of the goods sold in the past, he tends to lower the price in the following period(s). However, by rationally expecting the monopolist to behave in this way, customers would hold their purchases now and wait for the following period(s), thus resulting in the disappearance of the monopoly power “within a twinkling of eyes” (Coase 1972:143). The Coase conjecture is also called intertemporal externality or time inconsistency problem in the literature.

Then, why is Coase’ original example of land monopoly is largely ignored by both researchers in industrial organization and urban (land) economics? The reason is probably that land is a far more complex good than standard industrial outputs. Foldvary (1994) provided a clear analysis of “territorial collective good”, which indicates that land and local collective goods are bundled together. Not only is their consumption but their transactions are also bundled together (Deng 2002). The bundling of transaction is important because it rules out “home bundling;” consumers cannot buy land and territorial collective good separately and then consume them together at home. Therefore, we cannot separately consider the demand and
supply of the two goods; we have to consider the demand and supply of the bundle.

Nevertheless, the provision of land and collective goods can be separate, giving rise to many important institutional issues in urban land use (Deng 2003b). Although this unique feature of land complicates the modeling effort, it provides an important link between the Coase conjecture and urban institutions.

The motivation for this paper comes from several fronts. First, there has been widespread growth of private communities, especially in the suburbs (Barton & Silverman 1994; Gordon and Richardson 2001). Many have suggested various reasons for this worldwide phenomenon (Foldvary 1994; Blakely and Snyder 1999; Helsley and Strange 1998; Webster 2001; Deng 2003a). However, it is not clear how we can explain why they are mostly located in the suburbs. Besides, a typical concern from many commentators who criticize private communities is their alleged monopoly and consequent problems. Is monopoly really a big problem for private communities? What’s the relationship between land market structure and private communities?

Second, compared with the firm, one important feature of urban institutions is the dominance of public institutions.1 Many studies on Tiebout Hypothesis have found that some entrepreneurial behavior has to be assumed for local governments.2 Then, why couldn’t local governments take the form of profit-maximizing private entities? Obviously, existing studies in the vein of Tiebout Hypothesis can help us understand the behavior of local urban institutions but have difficulty in explaining their institutional forms.3 An extreme but theoretically legitimate question is: why couldn’t even higher-level governments, such as county or state, be

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1 In a penetrating study, Fischel (2001) analyzed American local government by comparing corporate voting and political voting. However, this approach doesn’t explicitly take into account the impact of market structure on institutional forms.

2 See Helsley (2003) for a good review on the studies related to local political institutions.

3 Recent important studies on local government and private communities (see Helsley and Strange 1998, 2000; Henderson and Thisse 2001) are largely in the vein of Tiebout model by focusing on the strategic interaction between private community and the public sector.
run as profit-maximizing private entities? Alternatively, in the spirit of Deng (2003a), why couldn’t the landowner and the provider of collective goods be integrated at large spatial scale? Since the territory of local government is usually quite big (when compared with most private communities), let alone higher-level governments, any theoretically possible private community at this spatial scale is essentially a monopoly in the land market. Hence, studies on endogenous urban institutions or even public institutions have to address the land monopoly issue.

Third, monopoly is an important issue in urban land use. Monocentric model has been at the central place of urban economics in the past decades. Its most important feature is the monopoly position of the city center and high differentiation of land in concentric rings. Obviously, high heterogeneity of urban land makes monopoly a useful model to understand various urban phenomena. On the other hand, based on the assumption of a competitive market, Tiebout (1956) hypothesis is often regarded as the benchmark model for local public services. It has been observed by many scholars that the two models are not well integrated (see, for example, Wheaton 1979). This paper presents an effort to integrate land market structure and the provision of local public goods.

Fourth, most contemporary studies on institutions and the firm focus only on the relationship among parties within a contract or organization (see, for example, Williamson 1985; Hart 1995), while those on property rights (Barzel 1989) largely hold a static view by focusing on *ex ante* property rights arrangements. By studying the impact of monopoly and intertemporal externality on urban institutions, this paper introduces both market structure and the time dimension into institutional studies. It provides a link between the literature on the Coase conjecture and that on urban institutions.
Lastly, many important land use issues involve intertemporal behavior of development interest versus existing residents. The change of population profile in the suburbs is probably one of the most important reasons behind NIMBY (Not in My Back Yard) and even urban sprawl (Fischel 1999). Many environmentalists are also arguing about the development impact on environmental quality and sustainable development, which is certainly related to intertemporal behavior. In this sense, this paper’s attempt to model intertemporal externality may help shed light on many important issues related to sustainable development.

There has accumulated a large body of literature on this topic in the economics of industrial organization.4 Bulow (1982) constructed a simple two-period model to analyze the problem faced by a durable-goods monopolist and how leasing can avoid it. The rational expectations equilibrium model in Stokey (1981) illustrates that as the length of the trading period approaches zero, the monopoly will eventually produce the competitive stock, following what Coase conjectured. Many studies also analyze the problem with some different assumptions and settings. Bond and Samuelson (1984) showed that depreciation and replacement sales reduce the monopolist’s tendency to cut price. Reducing the durability of the output can also help the monopolist to avoid the time-inconsistency problem (Bulow 1986). The choice of technology can also become an endogenous variable for the monopolist (Karp and Perloff 1996; Kutsoati and Zabojnik 2001). Some recent papers (Waldman 1996; Fudenberg and Tirole 1998) focus on technology upgrade and the interaction between the prices of new and used products.

The model setup in this paper is built on Fudenberg and Tirole (1998) that provides a general treatment of the time inconsistency problem. In this paper several features of the model reflect the uniqueness of land and the link to urban institutions. First, I explicitly introduce a

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4 See Waldman (2003) for a good review of related literature.
quality variable that stands for the collective good tied to land and, hence, is affected by how the collective good is provided—separately or bundled together. There are then four possible combinations of institutional settings depending on whether land is for rental or sale and whether the collective good is provided by the government or the monopolist: separate (provision of collective good) rental (land), bundled rental, separate sale, and bundled sale.

Second, in the case of separate provision of land and collective good, I assume a median voter model for the government provision of collective good. However, the treatment in the model is more general given the separability of consumer type and collective good quality in the utility function. In other words, the result applies to any model that only considers existing residents as long as the separability assumption holds.

Third, the collective good is not assumed to be durable and it has to be provided in each period. I also assume its provision has to cover all existing residents no matter they purchased land in the past or in the current period. Therefore, in addition to the intertemporal externality, there is also a typical “public good” externality. If the monopolist is also responsible for providing collective good, he cannot exclude existing residents who bought his land in the past from consuming the collective good. In the case of separate provision, the monopolist determines the scale of collective goods provision by selling or renting land while not being responsible for their provision. This free riding behavior of the land monopolist is certainly another source of externality.

The findings of the model show that, due to intertemporal externality, sales will result in more land development than rentals. Depending on the cost vis-à-vis WTP (Willingness to Pay), separate provision may result in lower quality of collective good in the second period. If only rental is possible, i.e., without intertemporal externality, separate provision will lead to more land
development due to the public good externality. A numerical example based on uniform
distribution of consumers suggests that separate provision of land and collective good may be
more efficient at large spatial scales. This can help explain the dominance of public institutions
at large spatial scales and why private communities are relatively small. In the numerical
example of a “poor” community, rental arrangements become more attractive, corroborating with
anecdotal evidence from historic company towns and current higher percentage of renters in
downtowns. In contrast, bundled sale, such as CID$s and condominiums, yields higher total
social surplus in a “rich” community. Overall, results from the numerical examples help to
explain why public institutions prevail in cities and most private communities are located in the
suburbs.5

The remainder of the paper is organized as follows. In the first section, the basic model
setup is introduced and the two benchmark cases of rental are discussed. The third section
constructs two-period models for the cases of sale with bundled and separate provision of
collective good, respectively. Then numerical examples based on different assumptions of
consumer distribution is provided to illustrate the difference between profit and social welfare
and the efficiency of different institutional arrangements in different circumstances. The last
section discusses the findings and raises issues for future research.

THE MODEL

A land monopolist is assumed to own the land that is demanded by a group of consumers.
The transaction and consumption of land and a collective good are always bundled together.

This is a fact for territorial collective good. The basic setup follows Fudenberg and Tirole

5 No doubt that the forces in Tiebout model are fundamental in the shaping of these different communities. But,
Tiebout model itself cannot explain their different institutional forms, especially regarding intertemporal externality.
(1998). There are two periods, \( t = 1, 2 \), and the discount factor for both the consumers and the land monopolist is \( \delta \). This world ends at the end of the second period. On the demand side, a continuum of consumers are indexed by \( \theta \in [0, 1] \), with a constant marginal utility of income. Per-period utility for a type-\( \theta \) consumer is \( \theta \cdot V(Q) + I \), where \( I \) is net income and \( Q \) denotes the quality of the bundle good—the provision of local collective good. \( \theta \cdot V \) is then equal to consumer’s valuation or willingness to pay (WTP) for the bundle good. Since the land is assumed to be infinitely durable and physically homogeneous, only the collective good may cause different qualities over time.

The distribution of consumer types on \([0, 1]\) is given by cumulative distribution function \( F(\cdot) \) with continuous density. Following Fudenberg and Tirole (1998), it is assumed that the hazard rate \( h(\theta) = f(\theta) / (1 - F(\theta)) \) is non-decreasing.\(^6\) Hence, given an existing stock of \( x_I \) of the bundle goods, potential or remaining consumers in the market for a price of \( \theta \cdot V \) are those indexed by \([\theta, 1]\) and their number is \( 1 - F(\theta) - x_I \). Also, quality \( Q \) is assumed to contribute positively to consumers’ utility or WTP, i.e., \( V'(Q) \geq 0 \).

On the supply side, the most important and obvious assumption is that land is bundled with a local collective good. Although their provision can be from different sources, their transactions and consumption are simultaneous.\(^7\) In the case of bundled provision, both goods are provided by the monopolist whose objective is profit maximization. In the case of separate provisions, we assume a median voter model for the provision of collective good while the supply of land is still determined by the land monopolist’s objective of profit maximization.

\(^6\) This assumption guarantees concavity of objective function. It also holds for any truncated demand function.

\(^7\) This fact makes the model a spatial model although it does not include traditional spatial variables (such as distance). This is distinct from ordinary industrial products.
Because land is infinitely durable, it is assumed that there is no cost of supplying land.\textsuperscript{8} The cost of supplying the collective good is assumed to be constant relative to spatial scale (constant return to scale) but change with $Q$, the quality variable.\textsuperscript{9} Denote unit cost as $C(Q)$ and assume $C'(Q) \geq 0$. In other words, the better is the collective good, the higher is the cost to provide it.

Since the literature on durable goods usually treats the rental case as the benchmark in which the time inconsistency problem can be avoided, I also present two benchmark cases of rental. In the first case, consumers rent land from the monopolist while some public entity (local government) determines how to provide the collective good. In the second benchmark case of bundled provision, both land and collective good are provided by the monopolist.

**Benchmark Case 1: Separate Rental**

With separate provision of land and collective good, the quality of the bundle is exogenous to the monopolist’s profit maximization problem. Given $Q_1$ and $Q_2$ in the two periods, the rental demand for the bundle goods at a rental price $r$ is equal to $1 - F(\theta)$, where

$$\theta \cdot V = r$$  \hspace{1cm} (1)

Then the optimization problem for the monopolist is to rent land to consumers with types above $\theta^*$ so that his profit can be maximized.

$$\text{Max. } [1 - F(\theta)] \cdot \theta \cdot V$$

\textsuperscript{8} It is also assumed that development cost, if any, is zero. A positive development cost will not change the basic results.

\textsuperscript{9} This assumption allows us to assume away the economy of scale without affecting basic results and focus only on collective good.
It is obvious that $\theta^*$ is independent of $V$. Therefore, in Period 1, the monopolist rents $1 - F(\theta^*)$ of $Q_1$ quality land at a rental price of $\theta^*V(Q_1)$; in Period 2, he rents the same amount of land to the same people at a price of $\theta^*V(Q_2)$. From the first order condition we can have

$$\theta^* = \frac{1 - F(\theta^*)}{f(\theta^*)} = \frac{1}{\text{HazardRate}}$$  \hspace{1cm} (2)

Because $\frac{1}{\theta h}$ is the inverse elasticity of demand, this equilibrium condition says that it has to be equal to one when the monopolist maximizes his profit. This is a standard result for a single-product monopolist when the relative “markup” —the ratio between profit margin and the price, which is also called the Lerner index—is equal to one given the assumption of no cost in supplying land. The intuition behind this condition is that the monopolistic price distortion from the marginal cost has to be balanced against the decline of consumers’ demand.

Since potential or future consumers haven’t rented land and are not living in this place, they are not included in the existing residents who determine the median voter. Given the separability of $\theta$ and $V$ in consumer’s utility, as specified in (1), all consumers’ utilities change proportionately with $V(Q)$. In this sense, maximizing the utility of the median voter, mean voter, or all voters does not make much difference to the result. The maximization problem for the government can be written as the following

$$\text{Max. } \int_{\theta^*}^{1} f(\theta)\theta V(Q)d\theta - \left[1 - F(\theta^*)\right]C(Q)$$

The first order condition with respect to $Q$ then becomes

$$\theta^* V'(Q) = \left[1 - F(\theta^*)\right]C'(Q)$$  \hspace{1cm} (3)

Here I slightly abuse the notation and use $\bar{\theta}^*$ to denote the mean of $\theta$ on $[\theta^*, 1]$. This condition means that the consumers’ marginal increase in valuation from higher quality of collective good
should be equal to the marginal cost of providing the collective good. It also suggests that the provision of collective good is stable over time in the case of separate rental, with the same quality $Q$ in each period.

**Benchmark Case 2: Bundled Rental**

With bundled provision, the monopolist provides both land and the collective good in a bundle and rents them to the consumers. He now faces the following maximization problem:

$$\text{Max. } [1 - F(\theta)] \cdot \theta V(Q) - [1 - F(\theta)] \cdot C(Q)$$

The first order condition then becomes

$$\text{Hazard rate } = \frac{f(\theta^*)}{1 - F(\theta^*)} = \frac{1}{\theta^* \left(1 - \frac{C}{V \theta^*}\right)}$$

$$\theta^* = \frac{C'(Q)}{V'(Q)}$$

The second condition (5) means that when maximizing the profit to the monopolist, the marginal increase of rent ($\theta^* V'$) should be equal to the marginal cost ($C'$) with regard to the collective good provision. The first condition (4) looks similar to (2) but with an extra factor, which leads to the following proposition.

*Proposition 1*: when only rental is possible, what the land monopolist rents in the case of bundled provision of land and collective good is less than or equal to that in the case of separate provision.

The fundamental reason behind Proposition 1 is the externality in providing the collective good. With separate provision of land and collective good, the monopolist can free-ride on the provision of collective good and doesn’t need to worry about the cost. In contrast, when he is
responsible for providing both land and collective good, he has to take into account the cost of providing the collective good. In the rental case intertemporal externality doesn’t exist. This is consistent with the literature on Coase conjecture.

Proposition 2: when only rental is possible, at equilibrium the first-order derivative of the ratio of cost vis-à-vis WTP (C/V) should be positive or equal to zero while the marginal cost has to be less than or equal to the marginal increase of WTP.

This proposition says that, on the one hand, in order to achieve equilibrium, the cost must increase faster than the consumer’s WTP. Otherwise, the land monopolist will be motivated to provide even higher quality of the bundle good—better provision of the collective good, possibly towards infinite if no boundary condition exists. Hence, equilibrium requires that the ratio of cost vis-à-vis WTP has to increase with quality of the bundle good. On the other hand, the marginal cost has to be no greater than the marginal increase of WTP. This ensures the monopolist’s profit is maximized.

In order to look at how bundling its provision with land affects the collective good, we can compare (3) and (5), the first-order conditions for the two benchmark cases. If we assume both cases have the same group of consumers renting the land, i.e., the same θ, then by the definition of θ we can have $\theta > \bar{\theta} > \theta$. It is then easy to see that separate provision, in which case the equilibrium requires the condition (3), has a higher $\frac{C'}{V'}$. This means that, ceteris paribus, separate provision can tolerate a relatively larger marginal cost in providing the collective good than bundled provision. This is not surprising given the free-riding behavior of the land monopolist.
TWO-PERIOD MODEL OF SALE

In many cases, leasing may be inefficient, impossible, or prohibited by the law and sale/purchase becomes the only or best option for the monopolist/consumers. Although leasing is usually regarded as good at avoiding the time consistency problem, many discussions in the literature of industrial organization have pointed out the problems of leasing (See, for example, Tirole 1988). In urban land use, MacCallum (1970) has been advocating for the leasehold-based proprietary communities. Deng (2002) argues that the leasehold-based system combines the efficiency properties of Tieboutian (1956) competition and George’s (1879) insight on rent capitalization. These arguments are all based on the assumption of a competitive market. If monopoly instead of a competitive market exists in the land market, then what is the impact of intertemporal externality on land development/sale and the provision of local collective good?

In order to model the time inconsistency problem, I assume the monopolist cannot make any commitment or guarantee in terms of future sales or prices. This may be more realistic for land use than for ordinary durable good because in the former case the monopoly is usually based on horizontal differentiation that might make pricing more difficult. An “anonymous” and frictionless second-hand market is also assumed to exist. Because the collective good is provided to all existing owners in the same period, it doesn’t cause any quality difference between new and old buyers that is typical in software and some other products (see Waldman 2003 and others). Because the second-hand market is frictionless, a consumer’s decision in the second period is not affected by whether or not he bought land in the first period. In other words, consumer doesn’t need to choose among used good and new good; they are treated the same in the model.
Assuming the quantity of land sold in the first period is $q_1$ with quality $Q_1$, I first work on the second period and then backwards on the first period. Of course, consistent with the literature, a key assumption is that consumers can correctly or rationally anticipate the second-period price in the first period.

**Bundled Provision**

I first assume that land and the collective good are provided in a bundle by the monopolist. He is motivated to provide them in a way that maximizes the profit.

**The second period.** In the second period, assume consumer of type $\theta_2$ is indifferent about whether to purchase land or remain outside the area. So, we have

$$P_2 = \theta_2 \cdot V(Q_2)$$

Consumers with $\theta$ above $\theta_2$ are all current residents in this area in the second period, including both those who buy land in the second period and those who bought a total amount of $q_1$ in the first period. That is, $I - F(\theta_2) = q_1 + q_2$. Hence,

$$q_2 = F(\theta_1) - F(\theta_2)$$

(6)

Now, the monopolist maximizes his second-period profit.

Max. $\Pi_2 = p_2q_2 - (q_1 + q_2) \cdot C(Q_2)$

$$= \theta_2 V(Q_2)[F(\theta_1) - F(\theta_2)] - [I - F(\theta_2)]C(Q_2)$$

(7)

The second term in the monopolist’s profit function is due to the assumption that the provision of collective good has to cover not only new buyers but also all existing residents.
This is based on the non-exclusivity assumption or the “public good” externality for the collective good within the territory. Then, the first order conditions become

\[
\frac{\partial \Pi}{\partial \theta_2} = V(Q_2)[F(\theta_1) - F(\theta_2)] - \theta_2 V(Q_2) f(\theta_1) + C(Q_2) f(\theta_2) = 0 \quad (8)
\]

\[
\frac{\partial \Pi}{\partial Q_2} = \theta_2 V'(Q_2) [F(\theta_1) - F(\theta_2)] - C'(Q_2) [1 - F(\theta_2)] = 0 \quad (9)
\]

**Proposition 3:** with bundled provision of land and the collective good, the monopolist has cumulatively sold more land up to the second period than he rents in the rental case.

This proposition states that, *ceteris paribus*, when land and the collective good are both provided in a bundle by the monopolist, the cumulative amount of land he sells up to the second period should be larger than the rental amount. Of course, the quantity of land the monopolist rents is the same in both two periods, given our discussion of the two benchmark cases. The reason for this result is that the existence of intertemporal externality allows the monopolist to expand the sales at the cost of the customers who bought land in the first period.

Rearranging (9), we can have

\[
\theta_2 \geq \frac{C'(Q_2)}{V'(Q_2)} \cdot \frac{q_1 + q_2}{q_2} \quad (10)
\]

Because \( \theta_2 \leq 1 \) and it is also smaller than \( \theta^* \) in the rental case (with bundled provision), as shown by Proposition 3, comparing (10) and (5) shows that \( \frac{C'(Q_2)}{V'(Q_2)} \) is now less than in the case of bundled rental if the second-period sale quantity \( \theta_2 \) is assumed to be the same.

Depending on the first-order derivatives of the cost and WTP, the impact of sale versus rental on the provision of collective good (the quality of the bundle good) can then be analyzed. For example, if the first order derivative of cost with regard to \( Q \) increases, i.e., cost increase faster
and faster, and the first order derivative of WTP decreases with $Q$, then (10) implies that bundled sale will result in lower quality (worse provision of collective good) given the same $\theta$.

Furthermore, since $\theta_2$ (the LHS in Eq. 10) is even smaller than in the rental case, as Proposition 3 shows, then the quality $Q$ will be even lower in this example and the effect on $Q$, as discussed above, will be even more significant.

**The first period.** We now work backward to the first period. As the rational expectation assumption implies, the land price in the first period should depend on their expectation of the second-period price. That is

$$p_1 = \theta_1 V(Q_1) + \delta \theta_2 V(Q_2)$$

(11)

So, the monopolist’s first-period profit function is

$$\Pi_1 = p_1 \cdot q_1 - q \cdot C(Q_1)$$

$$= \left[1 - F(\theta_1)\right] \cdot \left[\theta_1 V(Q_1) + \delta \theta_2 V(Q_2) - C(Q_1)\right]$$

(12)

Note the profit function subtracts the cost of providing the collective good because it is now bundled with land and enters the monopolist’s calculation. We can also obtain the second-period profit function as in (7). The monopolist then maximizes his overall profit

$$\Pi = \Pi_1 + \delta \cdot \Pi_2$$

$$= \left[1 - F(\theta_1)\right] \cdot \left[\theta_1 V(Q_1) + \delta \theta_2 V(Q_2) - C(Q_1)\right] + \delta \theta_2 V(Q_2) \left[F(\theta_1) - F(\theta_2)\right] - \delta C(Q_2) \left[1 - F(\theta_2)\right]$$

(13)

The first-order condition for $Q_1$ can be obtained as follows. The reason is that $Q_1$ is independent of $\theta_2$ and $Q_2$ given that the collective good is not durable and is always assumed to be provided to all residents within the area.

$$\theta_1 V'(Q_1) - C'(Q_1) = 0$$

(14)
The first-order condition for $\theta_i$ is quite complex involving the derivatives of $\theta_i$ with regard to $\theta_2$ and $Q_2$. In a simpler approach, we can directly use $q_1$, $q_2$ instead of $\theta_1$, $\theta_2$. Denote $x = q_1 = 1 - F(\theta_1)$, $y = q_1 + q_2 = 1 - F(\theta_2)$. Obviously, $\theta_2 = F^{-1}(1 - y)$. Also note that \(\left(F^{-1}\right)' = \frac{1}{f}, \left(F^{-1}\right)^{''} = -\frac{f''}{f^3}\). Then, the first-order conditions for the monopolist’s first-period profit maximization problem can be rewritten and the derivatives of $Q_2, y$ with regard to $x$ can be obtained.

\[
\frac{\partial Q_2}{\partial x} = \frac{f^2 - f'(y - x)}{U'(Q_2) \cdot f^3} \tag{15}
\]

\[
\frac{\partial y}{\partial x} = \frac{W'(Q_2) \cdot y \left[f^2 - f'(y - x)\right] + U'(Q_2) \cdot f^2 \left(y - x + f \cdot F^{-1}\right)}{f^3 \cdot U'(Q_2) \left[F^{-1} - W(Q_2)\right]} \tag{16}
\]

where $U(Q) = \frac{C(Q)}{V(Q)}$ and $W(Q) = \frac{C'(Q)}{V'(Q)}$.

To better understand the first-order conditions in (15) and (16), we can make some simplifying assumptions. If we assume $W'(Q_2) = 0$, i.e., the second-order effects of $C(Q_2)$ and $V(Q_2)$ are negligible, then from (16) we have

\[
\frac{\partial y}{\partial x} = \frac{\theta_2 V'}{\theta_2 V' - C'} \tag{17}
\]

So, given the linear assumption for the cost and valuation functions, if the marginal increase in WTP ($\theta_2 V'$) is higher than the marginal cost ($C'$), then $\frac{\partial y}{\partial x} > 0$. In other words, if the marginal price increase due to better second-period quality is larger than the marginal cost at equilibrium, then the standard result in durable goods literature still holds—total sale size will increase with the first-period sale. This result is consistent with the profit maximization goal of the monopolist. However, if the second-period quality is such that marginal revenue is less than...
marginal cost, i.e., $\theta_2 V' - C' < 0$, then $\frac{\partial y}{\partial x} < 0$, which means that the monopolist may even reduce the sale of land in the second period.\(^{10}\)

The meaning of (15) becomes clearer if we assume a uniform distribution function for the consumers. In that case, $f(\theta) = 1$, $F(\theta) = \theta$, $f'(\theta) = 0$, and (15) becomes $\frac{\partial Q_2}{\partial x} = \frac{1}{U'(Q_2)}$. So, if the ratio of cost vis-à-vis WTP increases with the quality of the bundle good, i.e., $U'(Q_2) > 0$, then $\frac{\partial Q_2}{\partial x} > 0$, meaning the quality of the bundle good, i.e., the provision of collective good, in the second period will increase with the quantity of sale in the first period. The more is sold in the first period, the better quality of collective good will the monopolist provide in the second period. Vice versa, if $U'(Q_2) < 0$, then $\frac{\partial Q_2}{\partial x} < 0$. In that case, the more land is sold in the first period, the worse quality of the bundle good will the monopolist provide in the second period. In general, the tradeoff is that, on the one hand, higher quality leads to higher cost, especially given the externality in providing the collective good, and on the other hand, higher quality results in higher price that could reduce intertemporal competition faced by the monopolist.

Now, in the monopolist’s second-period maximization problem, substitute (15) and (16) into the first order condition with regard to $x$ (like Eq. 8 and 9 that are first order conditions with regard to $\theta$) and we have

$$\theta_1 V(Q_1) - C(Q_1) - \frac{V(Q_1)}{h(\theta_1)} + \delta \left[ \frac{\theta_2 V(Q_2)}{h(\theta_2)} - \frac{V(Q_2)}{h(\theta_2)} - C(Q_2) \right] \frac{\partial y}{\partial x} = 0 \quad (18)$$

\(^{10}\) It is possible that the monopolist may even want to buy back land in order to save on the cost of providing the collective good.
where $h(\cdot)$ is the hazard rate function. Equations (14) and (18) can solve for $\theta_1$ and $Q_1$, given the relations between $\theta_2$, $Q_2$ and $\theta_1$ as specified in (15) and (16). Hence, the whole system is now mathematically solvable.

If we compare (bundled) sale with (bundled) rental, it is easy to see that the first-period quality is determined in the same way. But, the first-period quantities are determined very differently. Bundled sale in the first period depends also on what happens in the second period due to intertemporal externality.

**Proposition 4:** The relationship between quality and quantity in the first period are the same in the (bundled) sale and rental cases. If the second-order effects can be ignored and

$$\frac{\theta_2 V'(Q_2)}{h(\theta_2)} - \frac{V(Q_2)}{h(\theta_2)} - C(Q_2)$$

has different signs from $\frac{\theta_2 V'(Q_2)}{h(\theta_2)} - C'(Q_2)$, the first-period $\theta_1$ in the sale case is larger (i.e., smaller $q_1$) than in the rental case; otherwise, it is less than in the rental case.

This proposition basically describes how the (expected) second-period condition affects the first period sale quantity, as compared to the rental case. $\theta_2 V' - C'$ is the marginal increase of profit with regard to the quality. Rearranging $\frac{\theta_2 V(Q_2)}{h(\theta_2)} - \frac{V(Q_2)}{h(\theta_2)} - C(Q_2)$ shows that it is basically a comparison between the so-called relative “mark-up” $\frac{\theta_2 V - C}{\theta_2 V}$ and the inverse elasticity of demand $\frac{1}{\theta_2 h}$. Recall that in the (bundled) rental case, these two should be equal in the first-order condition (4). So, Proposition 4 states that, ceteris paribus, if increasing second-period quality also increases profit and, hence, the aggregate development scale over the two periods increases with the first-period quantity, then whether or not second-period profit margin
is less than what the monopolist charges in the rental case will determine if the monopolist will sell less quantity in the first period than he will rent. Profit margin being less than in the rental case is obviously good for the consumers, bad for the monopolist. Alternatively, if the aggregate development scale over the two periods decreases with the first-period quantity, then whether or not second-period profit margin is less than what the monopolist charges in the rental case will determine if the monopolist will sell more quantity in the first period than he will rent.

**Separate Provision**

Now consider the case that the land and the collective good are provided separately. The monopolist still maximizes profit from selling land to the consumers, but the collective good is provided by a government or some public entity that maximizes the (median voter) utility of the existing residents within the area.

**The second period.** The monopolist’s second-period profit function is

\[ \Pi_2 = \theta_2 V(Q_2) [F(\theta_1) - F(\theta_2)] \]

Obviously, he now doesn’t need to care about the cost of providing the collective good. The first-order conditions for the monopolist’s maximization problem are then

\[ F(\theta_1) - F(\theta_2) - \theta_2 f(\theta_2) = 0 \]

The government’s optimization problem is the same as in the rental case. In other words, first-order condition (3) holds here for the quality \( Q \) in both the first period and the second period.
The first period. Assume again that the consumers have correct or rational expectation in the first period and then Equation (11) still holds. The monopolist maximizes his overall profit

\[ \Pi = \Pi_1 + \delta \Pi_2 \]
\[ = [\theta_1 V(Q_1) + \delta \theta_2 V(Q_2)] - [1 - F(\theta_1)] + \delta \theta_2 V(Q_2) \left[ F(\theta_1) - F(\theta_2) \right] \]

(21)

To simplify the solution, we still use \( x, y \) to replace \( \theta_1 \) and \( \theta_2 \), and then the first-order condition for the maximization problem in the second period, i.e., Eq. (20), becomes

\[ F^{-1}(1 - y) - \frac{y - x}{f(\theta_2)} = 0 \]

(22)

Taking derivative with regard to \( x \) yields

\[ \frac{\partial y}{\partial x} = \frac{1}{2} \left( y - x \right) \frac{f''}{f^2} + \frac{1}{2} \]

(23)

Because of the assumption of monotone hazard rate, \( f' > 0 \). Hence, (23) implies \( \frac{\partial y}{\partial x} > 0 \), which is consistent with the standard result in the durable goods literature.

The first order condition for the first period maximization problem is now

\[ \theta_1 V(Q_1) - \frac{V(Q_1)}{h(\theta_1)} + \delta \left[ \theta_2 - \frac{1}{f'(\theta_2)} \right] - V(Q_2) \frac{\partial y}{\partial x} = 0 \]

(24)

Now the system that consists of (3) (for both \( Q_1 \) and \( Q_2 \)), (22), (23), and (24) is mathematically solvable. Although the first order relationship between \( Q \) and \( \theta \) in the (separate) sale case is the same as in the rental case, as specified in (3), the change of the aggregate sale quantity \( y \) (or the decrease of \( \theta \) in the second period) means the provision of the collective good will also change.
Proposition 5: in the case of (separate) sale, if there is no negative sale (buy-back) in the second period, the change of quality of the bundle good from the first period to the second period depends on how \( \frac{C'(Q)}{V'(Q)} \) decreases. If marginal cost increases faster than marginal WTP with regard to \( Q \), then quality (provision of the collective good) declines in the second period; if marginal cost increases slower than WTP, then quality rises in the second period.

Because the collective good is now provided by the government, its provision is affected by current population that in turn depends on the cumulative quantity of land sold. If we assume that, in net, the monopolist doesn’t buy back land in the second period, the cumulative land quantity increases over time. Hence, median voter’s valuation of the bundle good decreases with growing population. \( \frac{C'(Q)}{V'(Q)} \) measures the marginal cost vis-à-vis marginal utility. Equilibrium condition requires it also declines from the first period to the second period. So, Proposition 5 basically states that if cost increases faster than utility or WTP, the second-period government (or the median voter) will opt for lower quality of the collective good; otherwise, the quality will rise.

**SOCIAL WELFARE AND NUMERICAL EXAMPLE**

To analyze which institutional setting is best for either the society or the monopolist, we can calculate total social surplus and monopoly profit in each case and then compare them. The institutional arrangement that yields the highest social surplus is most efficient from a social perspective. Because the quantity and quality of the bundle goods sold in each period are all interdependent, it is difficult to obtain closed form solutions without specifying the functional
forms. Numerical methods are used in this section to provide examples that illustrate how different distributions of consumers affect the most efficient form of institutions.

If we assume “social” discount rate is the same as the one used in the rational expectation of price, then we have the following formula to calculate total social surplus:

\[ S = \bar{\theta}_1 V(Q_1) - \left[ 1 - F(\theta_1) \right] \cdot C(Q_1) + \delta \bar{\theta}_2 V(Q_2) - \delta \left[ 1 - F(\theta_2) \right] \cdot C(Q_2) \] (25)

where \( \bar{\theta}_1 = \int_0^1 \theta f(\theta) d\theta \) and \( \bar{\theta}_2 = \int_0^1 \theta f(\theta) d\theta \). This definition basically aggregates all current residents’ utility minus the cost of providing the collective good in each period and then adds the discounted second-period value to the first period one.

For simplicity, the discount rate is assumed to be one, i.e., no discount. We also assume the following functions for the cost of providing the collective good and the consumers’ valuation (or WTP).

\[ C(Q) = Q^2 \]

\[ V(Q) = 2 + 2Q \]

Given the importance of consumer distribution to the model, three different distribution functions are assumed for the following three numerical examples, respectively (Figure 1). For the first numerical example, a uniform distribution of consumers is assumed on \([0, 1]\). Then,

\[ F(\theta) = \theta, \quad f(\theta) = 1, \quad f'(\theta) = 0, \quad \bar{\theta} = \int_0^1 \theta f(\theta) d\theta = \frac{1}{2} \left( 1 - \theta^2 \right) \]. In the second example, the density function is assumed to be linearly decreasing to zero when \( \theta \) equals one. \( F(\theta) = 2\theta - \theta^2 \),

\[ f(\theta) = 2 - 2\theta, \quad f'(\theta) = -2, \quad \bar{\theta} = \frac{1}{3} + \left( \frac{2}{3} \theta^3 - \theta^2 \right) \]. This is like the case of a “poor” community where consumers are concentrated in lower end of WTP distribution. The third example is the

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11 I assume a myopic government who only cares about existing residents in the current period. This assumption is obviously different from maximizing total social surplus as in (25).
opposite, a “rich” community, where consumers are concentrated in the upper end of the distribution. The density function is a linear increasing function of $\theta$ that starts from zero when $\theta$ equals zero. $f(\theta) = 2\theta$, $F(\theta) = \theta^2$, $f'(\theta) = 2$, $\overline{\theta} = \frac{2}{3}\theta^\frac{3}{2}$.

[Figure 1 around here]

With these specifications of the functional forms, we can then solve the problems in different institutional settings either directly or by using numeric methods. Table 1 lists the results for the three examples, each of which include four different institutional arrangements.

[Table 1 around here]

There are some general results that largely hold for all three examples. First, the two rental cases have the same land quantity and quality in the two periods. This is expected because rental can effectively avoid the time inconsistency problem. Second, bundled rental (proprietary community) generally results in more development than separate rental due to the public good externality. In the cases of uniform distribution and “poor” community, bundled rental has lower quality of collective good than separate rental. But, in the case of “rich” community, bundled rental yields higher quality of public goods. Third, with separate sale, more land is provided in the second period and the provision of collective good (the quality) declines in the second period. This appears to fit well into the common perception of real estate development, especially its impact on public goods and public services. But, with bundled sale (like in CID), the size of land development actually decreases in the second period with a sharp increase in the collective good quality. This result from the numerical examples implies that the monopolist actually buys back land while providing higher quality of collective good in the second period.

It is obvious from Table 1 that, in the example of uniform distribution, separate sale has the highest value of both profit and total social surplus. This result supports the common form of
local government provision of collective goods in urban areas (Fischel 2001). On the one hand, sale or homeownership makes intertemporal price competition possible, which weakens the monopoly power and is therefore good for consumers. On the other hand, separate provision also deprives the monopolist of bundling as a way to weaken intertemporal competition. In a sense, this numerical example helps to explain why public institutions prevail at large spatial scales, where consumers are more evenly distributed, and why most private communities are relatively small-scale.

In the “poor” community example, separate sale also dominates other institutional arrangements. However, two rental arrangements yield profit and social surplus that are close to separate sale and significantly higher than bundled sale. Specifically, separate rental has almost the same value of social surplus as separate sale. The reason may be that the concentration of consumers at the lower end of the distribution reduces the difference between sale and rental in terms of total social surplus. The monopolist’s profit in bundled rental is also close to that of separate sale. All these results indicate the attractiveness of rental arrangements, corroborating with anecdotal evidence that poor neighborhoods have more renters, especially in downtowns. Bundled rental in company towns might also be a case in point.

In contrast, the “rich” community example shows that arrangements with bundled provision of land and collective goods can yield highest social surplus or profit. Bundled sale has the highest social surplus and bundled rental generates highest profit. The concentration of high-valuation consumers provides only limited opportunity for the monopolist to reduce intertemporal competition. By doing so, the monopolist has to significantly lower the price, increase the sales in the second period and consequently allow more consumers to satisfy their demand, resulting in higher total social surplus. This example supports the common observation

\[12\] Of course, affordability is a real-world constraint for low income households.
that most private communities are built for middle-upper class people and are mostly in the form of CID's or condominiums, both of which are essentially "bundled sale" based on homeownership.

In summary, the three numeric examples demonstrate the importance of consumer distribution to the efficient institutional arrangement. More uniform distribution of consumers, such as at large spatial scales, makes separate sale more efficient. Rental is more attractive for the community with a concentration of "poor" consumers. Integrated provision of land and collective goods, so-called private communities, may be closely related to the concentration of "rich" consumers.

DISCUSSION AND CONCLUSION

By focusing on intertemporal externality in the market of a bundle good of land and territorial collective good, the two-period model developed in this paper revisits the Coase conjecture in its original example of land monopoly. The building block is that the transaction and consumption of land and collective good are bundled together but their provision can be separate. This fact provides the link among land monopoly, intertemporal externality, and urban institutions.

The findings point to the importance of intertemporal externality in urban land use. In a world of only rentals, separate provision of land and collective good results in more land developed. This suggests that, ceteris paribus, proprietary communities that is based on leasehold and bundles the provision of land and collective good may result in less development than leasehold under traditional government provision of collective good. In the case of bundled rental, the quality variable that stands for the collective good largely depends on how cost and
WTP change relative to each other. Given the same quantity of land rented, separate provision
can tolerate a larger cost increase in the provision of the collective good.

In the case of sales, the interactions among land quantity and quality in the first and
second periods become more complicated. With bundled provision of collective good, a larger
cumulative quantity of land will be sold than what could be rented. Since non-durable collective
good is bundled with land in their transaction and consumption, intertemporal competition is
weakened and the monopoly power is strengthened. The result is more (or over) development.
In the case of separate provision, collective good provision deteriorates in the second period if
cost increases faster than WTP with regard to the quality, and vice versa if WTP increases faster
than cost.

The model not only provides insight into the dynamics of urban land use under
monopoly, but may also be helpful in explaining many important land use phenomena. For
example, with certain distribution of consumer types, private communities may provide better
quality of collective good. In contrast, with separate provision, developers are more willing to
build low-income housing than local government in order to exploit both intertemporal
externality and public good externality. Leasehold is also an effective way to avoid
intertemporal externality and provide a stable flow of collective goods over time.

Moreover, findings from the model have important institutional implications about the
relationship among urban spatial structure, urban institutions and local collective goods
provision. Suburbs obviously face more competition than the city center, which enjoys
significant monopolistic power. My analysis and the numeric examples suggest that, within
some range of parameterization (such as uniform distribution of consumers), monopoly may
cause separate provision (i.e., government provision of collective good) more desirable. The
reasons for this result include (1) sale or homeownership creates intertemporal externality that is good for consumers; (2) bundling is instead good for the monopolist because it weakens intertemporal competition and emboldens time inconsistent behavior. This might be one reason why most private communities are located in the suburbs. A plausible explanation for the growth of private communities in the suburbs is that they are a new institutional form and most new developments are in the suburbs. Therefore, most private communities are located in the suburbs. However, this argument does not hold in the case of urban renewal, and private communities such as company towns did exist in the history (Fishback 1992). In this sense, this paper provides a competing hypothesis for the location choice of private communities. To some extent, this argument may also help to alleviate many people’s concerns about monopoly in private communities because these institutional forms may only be able to thrive in a more competitive market.

The model also helps to explain why private communities, in the form of the integration of landowner and collective goods provider, remain at small scale while public institutions prevail at large spatial scales. The first numerical example shows that intertemporal behavior of land monopolist in providing collective good is an important problem. Separate provision, or public institutions, can effectively mitigate this time-inconsistency problem at large spatial scales where consumers are more evenly distributed. Given that most public institutions, such as various levels of the government, are also territorial institutions, this argument sheds light on the dominance of public institutions that conventional wisdom often takes for granted.

Numeric examples also demonstrate important relations between consumer distribution and efficient institutional arrangement. With a high concentration of “poor” consumers, such as

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13 Fishback’s (1992) analysis indicates the important role of monopoly for company towns, where profit maximization as an objective certainly overrode other social objectives.
in downtowns or company towns, separate or bundled rental can be quite efficient institutional forms. In contrast, a high concentration of “rich” consumers may make bundled sale (such as CIDs and condominiums) more efficient. It is then not surprising that private communities are often characterized as “secession of the successful” (Reich 1991).

A key assumption in the model is that consumers can correctly anticipate the price in the second period. This is of course a strong assumption in land use, given the high heterogeneity of urban land. If this rational expectation assumption is relaxed, intertemporal competition will certainly become more weakened and the monopoly power will be strengthened. The result might be more sales (overdevelopment) in the second period, lower quality of collective goods, and then more conflict between the development interest and current residents.

There are many interesting issues deserving future research. First, the model is built on the assumption of a fixed group of consumers with a land monopolist. It will be interesting to explore other incomplete competition models and more explicitly incorporate the Tieboutian competition. Second, the model doesn’t explicitly take into account how local public goods are financed by government in the case of separate provision. The next step should be to explicitly incorporate property tax and allow rent capitalization, which is in turn related to the migration of consumers. Third, given the variety of collective goods, it is important to analyze the different impacts of different types of collective good. For example, some collective goods, such as transportation facilities, are durable. Durable collective good may actually weaken the monopoly power. Also, some collective goods may be congestible, especially after reaching some threshold. For example, environmental quality can be regarded as a collective good; it is

14 The local government’s maximization problem in the model implicitly assumes a property tax proportional to the median voter’s land value, which is equivalent to his utility besides income. In this sense, the behavior assumption of local government is similar to Brueckner (1983) and Fischel (2001).
then probably negatively related to land development or total population. This is certainly important to sustainable development, which is essentially about intertemporal issues over time.
APPENDIX

Proposition 1: when only rental is possible, what the land monopolist rents in the case of bundled provision of land and collective good is less than or equal to that in the case of separate provision.

Proof: In the case of bundled provision of land and collective good, since hazard rate $= \frac{1}{\theta^*\left(1 - \frac{C}{V\theta^*}\right)}$ and $\theta \in [0,1]$, obviously $1 - \frac{C}{V\theta^*} \geq 0$ or else hazard rate would become negative.

Given that the hazard rate and $\theta$ are both within $[0, 1]$, we must have $0 \leq 1 - \frac{C}{V\theta^*} \leq 1$. Hence, we have

$$\frac{1}{1 - \frac{C}{V\theta^*}} \geq 1 \quad (A1)$$

Because hazard rate and $\theta$ are both assumed to be non-decreasing, $(\theta \cdot \text{hazard rate})$ is also non-decreasing. Comparing condition (2) and (4), we can see that $(\theta \cdot \text{hazard rate})$ is bigger than 1 with bundled provision while it is equal to 1 with separate provision. Therefore, $\theta$ is not smaller in the case of bundled provision and the rental quantity $q = 1 - F(\theta)$ is then not larger.

Q.E.D.

Proposition 2: when only rental is possible, at equilibrium, the first-order derivative of the ratio of cost vis-à-vis WTP $(C/V)$ should be positive or equal to zero while the marginal cost has to be less than or equal to the marginal increase of WTP.
Proof: Transforming (A1) yields \( \frac{C}{V\theta} \leq 1 \), and hence \( \theta' \geq \frac{C}{V} \). Comparing with (5) then yields \( \frac{C'}{V'} \geq \frac{C}{V} \). Given that \( V' \geq 0 \) and \( V \geq 0 \), we have \( CV' \geq CV' \). Thus, \( \left( \frac{C'}{V'} \right) \geq 0 \). Because \( \theta \in [0, 1] \), condition (5) also implies that \( 0 \leq \frac{C'}{V'} \leq 1 \).

Q.E.D.

**Proposition 3:** with bundled provision of land and the collective good, the monopolist has cumulatively sold more land up to the second period than he rents in the rental case.

Proof: Rearranging the terms in (8) and (9) and then dividing them yields

\[
\frac{V(Q_2)}{\theta_2 V'(Q_2)} = \text{hazard rate} \cdot \frac{\theta_2 V(Q_2) - C(Q_2)}{C'(Q_2)}
\]

Substituting (9) into (A2) we can have

\[
\text{hazard rate} = \frac{V(Q_2)}{\theta_2 V(Q_2) - C(Q_2)} \cdot \frac{F(\theta_1) - F(\theta_2)}{1 - F(\theta_2)} = \frac{V(Q_2)}{\theta_2 V(Q_2) - C(Q_2)} \cdot \frac{q_2}{q_1 + q_2}
\]  

(A2)

Because \( \frac{q_2}{q_1 + q_2} < 1 \), \( \frac{1}{1 - \frac{C}{V\theta}} \) is a decreasing function of \( \theta \), and \( (\theta \cdot \text{hazard rate}) \) is a non-decreasing function, then comparing (A2) and (4) shows that \( \theta_2 \) is smaller now than in the rental case (also with bundled provision).

Q.E.D.

**Proposition 4:** the relationship between quality and quantity in the first period are the same in the (bundled) sale and rental cases. If the second-order effects can be ignored and
\[ \theta_2 V(Q_2) - \frac{V(Q_2)}{h(\theta_2)} - C(Q_2) \] has different signs from \[ \theta_2 V' - C' \], the first-period \[ \theta_2 \] in the sale case is larger (i.e., smaller \[ q_2 \]) than in the rental case.

Proof: It is easy to see that the first-order conditions (5) and (14) are essentially the same. If the second order effects can be assumed to be ignorable, substituting (17) into (18) and rearranging the items yields

\[
\theta_1 = \frac{1}{V(Q_1)} \left[ C(Q_1) + \frac{V(Q_1)}{h(\theta_1)} - \delta \left( \theta_2 V(Q_2) - \frac{V(Q_2) - C(Q_2)}{h(\theta_2)} \right) \cdot \frac{\theta_2 V'}{\theta_2 V' - C'} \right] \quad (A3)
\]

Obviously, (A3) is exactly the same as (4) except the last item, which is positive if \[ \theta_2 V(Q_2) - \frac{V(Q_2)}{h(\theta_2)} - C(Q_2) \] and \[ \theta_2 V' - C' \] have different signs. Given the non-decreasing assumption of the hazard rate, the sum of the first two items in (A3) is non-increasing with regard to \[ \theta_1 \]. By comparing (A3) and (4), it is then easy to see that \[ \theta_1 \] will be larger in the (bundled) sale case than in the (bundled) rental case.

Q.E.D.

**Proposition 5:** in the case of (separate) sale, if there is no negative sale (buy-back) in the second period, the change of quality of the bundle good from the first period to the second period depends on how \[ \frac{C'(Q)}{V'(Q)} \] decreases. If cost increases faster than WTP with regard to \( Q \), then quality (provision of the collective good) declines in the second period; if cost increases slower than WTP, then quality rises in the second period.

Proof: Transforming (3) yields
\[ \frac{\bar{\theta}^*}{1 - F(\theta^*)} = \frac{C'(Q)}{V'(Q)} \]  \hspace{1cm} (A4)

It’s then obvious that when \( \theta \) decreases from \( \theta_1 \) to \( \theta_2 \), the left-hand side of (A4) becomes smaller, meaning that \( \frac{C'(Q)}{V'(Q)} \) has to decrease in the second period.

\textit{Q.E.D.}
Figure 1: Consumer Distributions in the Numerical Examples

- Uniform Distribution
- "Poor" Community
- "Rich" Community
### Table 1: Results of the Numerical Examples

<table>
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<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$Q_1$</th>
<th>$Q_2$</th>
<th>Profit</th>
<th>Social Surplus</th>
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REFERENCES


