FINANCIAL DEVELOPMENT AND PROPERTY VALUATION

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ABSTRACT

This paper investigates the impact of financial development on property valuation in a rational expectations framework by modeling the agency theoretic perspective of risk averse investors (property owners) and financiers (banks/ capital markets). In contrast to previous research, we consider a setting in which financiers possess no inherent information processing or monitoring advantages. We demonstrate that property financing is undertaken in a pecking order of increasing pareto-efficiency (with reduction in its overall costs and a subsequent increase in the value of the underlying collateral) in a three staged process as financial architecture advances from a partially liberalized bank to the developed stage of capital markets. The primary solution is obtained in the rudimentary stage of commercial banks (in a specialized banking system), where the default-free mortgages are pareto-optimal to defaulting mortgages in accordance with the prognosis of Scott (1976) and Stulz and Johnson (1985). A pareto-improvement of the first solution is obtained by removing the restriction on ownership of property for financiers such as universal banks and pension funds, insurance companies, etc. This solution resolves the real estate version of the asset location puzzle (see Geltner and Miller, 2001). A further pareto-enhancement of this equilibrium is obtained under financial innovation by embedding the above default-free mortgage with options (in the form of a participating mortgage) in accordance with the prognosis of Green (1984), Haugen and Senbet (1981, 1987) and Schnabel (1993). Our results yield implications for financial system development. Our analysis predicts that an optimal financial system will configure itself skewed towards capital markets irrespective of the source of its origination (from specialized banking system or universal banking system). We also rationalize the co-existence of banks and financial markets in a well-developed financial system.

JEL Classification Codes: D58 (Computable and Other Applied Gen. Equilibrium Models)  
                            G12 (Asset Pricing)  
                            G2 (Financial Institutions and Services)  
                            G32 (Financing Policy; Capital and Ownership Structure)  
                            O16 (Financial Markets; Saving and Capital Investment)  
                            R13 (Welfare Analysis of Regional Economies)

Key Words: Financial Deepening; Financial Innovation; Financial Liberalization; Pareto-optimal Mortgage Design; Risk Management.
I. INTRODUCTION

The recent Asian financial crisis illustrates how fragile financial systems can devastate a country's real estate sector and thus subsequently its economy. In addition to its severe effects in Asia, the crisis had put pressure on emerging markets outside the region; contributing to virulent contagion and volatility in international financial markets. According to Lane (1999), the crisis mainly stemmed from inherent weaknesses in financial systems.1 The Asian crisis brings to focus the vital linkage between the financial system of a country and value of its assets.

This fragility of the financial system has been debated by financial economists and policy makers in the context of the role of banks and capital markets in mobilizing resources and enhancing economic growth. The success of capital markets in the common law countries (English speaking US and UK etc.) have led some observers to tout their virtues, while others have advocated financing by banks because of their vital role in German and Japanese industrialization.2

Researchers concerned with institutions and economic development seek to understand the dynamic process by which institutions evolve and interact with the rest of the economy. This paper analyzes the evolution of the financial system, with the objective of isolating factors that shape its development and long-run character. Much of the discussion in financial system development relates to Gerschenkron's (1962), who deduced (from empirical evidence of Germany, Italy, Russia and U.K.) that bank prominence in economic development stems from economic backwardness. When economic growth accelerates, market finance gradually replaces bank finance. Thus, capital markets play a prominent role in the financial sector of highly developed countries. There are three major channels through which a well developed financial system (capital market dominated) is welfare enhancing (e.g., see Pagano, 1993 and Levine, 1997). First, the provision of financial services can

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1 Lane (1999) explicates the crisis to three related issues. First, borrowing in foreign currencies without adequate hedging which reflects poor use of risk management strategies. Second, substantial rise in the prices of equity and real estate markets in these countries before the crisis, increasing the likelihood of a sharp deflation in asset prices. Third, poor credit allocation by the banks, contributing to increasingly visible problems at banks before the crisis hit. These factors reflected ineffective financial supervision and regulation in the context of countries' financial sector liberalizations.

encourage the mobilization of savings from many disparate savers. Financial systems enhance welfare by improving the efficiency with which those savings are used and increasing the allocation of funds to firms, thereby facilitating the growth of capital and productivity. That is, efficient financial systems can raise firm investment by reducing liquidity risk and idiosyncratic risk. Moreover, by mitigating risk (particularly liquidity risk), financial systems positively affect economic growth, since they eliminate the premature liquidation of firm capital. Second, well-developed financial systems (particularly capital markets) resolve agency problems better (Boyd and Prescott, 1986). This enables firms to borrow at cheaper rates and invest more. For example, capital markets enhance corporate control by (i) aligning the interests of manager-entrepreneur with those of lenders, and (ii) facilitating takeovers to mitigate the principal-agent problem (agency costs) and so encourage economic welfare. Therefore, a financial system develops to take care of agency problems. Third, improvements in risk-sharing can enhance savings rates and promote innovative, high-quality projects. For example, capital markets reduce liquidity risk by allowing agents who receive liquidity shocks to readily and cheaply sell their shares in firms. Similarly, banks mitigate liquidity risk by issuing demand deposits and by pooling savings of individuals.

In recent years, policymakers have been advocating a shift toward capital markets, particularly in Latin America and Eastern Europe where financial systems similar to those in the common law countries have been proposed (Allen and Gale, 2000). It is, however, unclear how financial systems will evolve in the future and what the welfare implications of this evolution are likely to be. As Levine (1997, pp. 702-703) points out, "we do not have adequate theories of why different financial structures emerge or why financial structures change. We need models that elucidate the conditions, if any, under which different financial structures are better at mitigating agency costs." It is precisely here that the chief contribution of our paper lies.

It is almost an article of faith now that the primary reason for existence of banks is the resolution of pre-contract private information or post-contract moral hazard problems. This
insight originated with Leland and Pyle (1977), followed by Diamond (1984), Ramakrishnan and Thakor (1984), Allen (1990), and Boot and Thakor (1997). Empirical support for these theories appears in Lummer and McConnell (1989), and others. This paper challenges this information-based paradigm and provides an alternative theory that does not require the financier to have any information-processing or monitoring advantage. While the information-based theories have served us well, recent events present us with a bit of a conundrum and suggest a need for rethinking. Advances in technology have led not only to much more public availability of financial information, but also a greater availability of tools with which to analyze it. This raises a question about the comparative advantage of banks in analyzing credit risks and resolving pre-contract private information problems, and there is reason to doubt that bank financing resolves informational problems at lower cost than possible with capital market financing (Coval and Thakor, 2004). Instead, our model suggests some advantages of capital market financing. We identify, financial liberalization, financial deepening, risk management and financial innovation as the key transmission channels of financial system development, as financing advances from (i) banks to capital markets and (ii) plain vanilla debt to innovative ones such as participating debt. Therefore, increased capital market sophistication and presence of non-bank financiers in capital markets diminishes bank lending (See Figure 1).

Abiad et al (2004) refer to financial liberalization as a reduction in the role of government and an increase in the role of the market in allocating credit. The indicators often used for it in the empirical literature are credit controls, interest rate controls, entry barriers for banks, regulations and restrictions on international financial transactions. McKinnon (1973) and Shaw (1973) propound the financial liberalization hypothesis arguing that government restrictions on the banking system restrain the quantity and quality of investment. In a repressed financial system real interest rates are kept artificially low (by the government). Financial development fails because the real return on bank deposits is too low.

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4 A variety of loans are now analyzed using neural networks and standard non-proprietary credit-screening programs that are not the exclusive domain of banks (see Greenbaum and Thakor, 1995).
Solution 1
Partially Liberalized Commercial Bank
- Financial Deepening
- Risk Sharing

Solution 2
Universal Banks/Non-Bank Financiers
- Financial Innovation

Solution 3
Capital Markets
- Further Financial Deepening

Figure 1: Financial System Development

Rudimentary Stage
(Bank Dominated)

Developed Stage
(Capital Market Dominated)

<table>
<thead>
<tr>
<th>Stage of Financial System Development</th>
<th>Rudimentary Stage (Bank Dominated)</th>
<th>Developed Stage (Capital Market Dominated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Markets</td>
<td>Small</td>
<td>Large</td>
</tr>
<tr>
<td>Liquidity*</td>
<td>Less liquid</td>
<td>Highly liquid</td>
</tr>
<tr>
<td>Risk-sharing</td>
<td>Intertemporal</td>
<td>Cross-sectional</td>
</tr>
<tr>
<td>Presence of Non-bank Financiers</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Financial Innovation</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Ownership and Control</td>
<td>Concentrated</td>
<td>Dispersed</td>
</tr>
<tr>
<td>Market of Corporate Control</td>
<td>Hostile takeover rare</td>
<td>Hostile takeover frequent</td>
</tr>
<tr>
<td>Dominant Agency Costs</td>
<td>Underinvestment problem</td>
<td>Underinvestment and Risk-Shifting problem</td>
</tr>
<tr>
<td>Role of Banks in External Finance</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Government Regulations**</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Loan-to-Value Ratio**</td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>

* In a partially liberalized commercial bank, there are no restrictions on LTV ratio, but the bank cannot hold equity positions in properties (a regulatory constraint).
* This is a parameter for degree of financial deepening
** These are parameters for degree of financial liberalization

Figure 1: Financial System Development
or even negative. The limited amount of available loanable funds is typically rationed (in accordance with government directives) reducing the quality of investment. Thus, economic growth suffers as both the quantity and quality of investment are low. In contrast, a liberalized financial system (dominated by capital markets – with no restrictions on direct ownership of assets) leads to market determined interest rates resulting in efficient allocation of capital (credit). This implies that a liberalized financial system is in a better position to promote economic growth and development than a repressed one (Arestis and Demetriades, 1996). In other words, financial liberalization enhances the social welfare.

We narrow our focus to the government regulation of banks. It is a well known fact that banks in the U.S. are restricted from taking equity positions in properties (Greenbaum and Thakor, 1995). According to Stulz (2000) and Allen and Herring (2001), allowing banks to hold equity positions in assets has advantages as well as disadvantages. A bank that takes an equity position in an asset (along with debt) cares more about overall asset value than one that does not. However, having banks hold equity exposes them to systemic risks. That is, makes them more vulnerable to financial crises that would damage the financial system to such an extent that economic activity in the wider economy would suffer.5 We initially consider the case of a partially liberalized commercial bank, which has no restrictions on its LTV ratio, but is prevented from holding equity positions in firms.

Abiad et al (2004) refer to financial deepening as the increase in the volume of credit being intermediated in financial markets.6 Many recent studies use the term financial development and financial deepening interchangeably. However, we consider financial development to be much broader in scope encapsulating financial liberalisation, financial deepening, risk management and financial innovation. We call these the four pillars of financial development as they impact on the efficiency of financial system. In a liberalized

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5 The Asian crises of 1997 occurred in economies where bank financing was dominant. The non-contingent nature of banks obligations appeared to play an important part in causing the crises. The primary aim of banking regulation is to prevent crises. It failed to do so in Asia. This has led to the suggestion that a move towards a liberalized financial system (capital market dominated) would be desirable in such economies.

6 The distinction between financial liberalization and deepening is not often made in the literature.
financial system (capital market dominated), financial deepening occurs with increase in funds allowing a greater volume of investment to take place through capital markets. Economies that have financially deep markets have high capital market liquidity which increases the intrinsic value of assets traded in it.\(^7\)

According to Allen and Gale (1997), a well developed financial system (capital market dominated) has a comparative advantage in providing *cross-sectional* risk-sharing (i.e., diversification of risk at a given point in time). This is due to the presence of financiers who are not constrained from taking equity position in firms (dispersed ownership).\(^8\) The Allen and Gale (1997) theory thus predicts that as financial system moves towards developed stage (capital market dominated), risk management through the use of derivatives and other similar techniques will become more important. Hence financial innovation plays a very important role in the risk management process (strategy and tactics) of organizations. The theory is thus consistent with the fact that risk management techniques are important in economies with developed financial systems (capital market dominated) than in rudimentary financial systems.\(^9\)

This view is also supported by Levine (2002), who argues that capital markets provide a richer set of risk-management tools that permit greater customization of risk ameliorating instruments (like participating mortgages, convertible mortgages, etc.). While banks may provide inexpensive, basic risk management services for standardized situations, a well developed financial system (capital market dominated) provides greater flexibility to tailor made products. Thus, as economies mature and need a richer set of risk management tools and vehicles for raising capital, they may concomitantly benefit from an environment that supports the evolution of capital markets.

Financial innovation improves *efficiency* of a financial system by reducing *endogenous* agency costs of debt (Allen and Gale, 1994 and Merton, 1990, 1995). When firms are debt

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8 Banks may ease the *intertemporal smoothing* of risks that cannot be diversified at a given point in time. In capital markets, on the other hand, intertemporal smoothing is ruled out by competition (see Allen and Gale, 1997, 2000).

9 This view is also supported by Levine (2002).
financed, manager-entrepreneurs have an incentive to transfer downside risk (of project) to the financiers while benefiting from the upside potential. This is a well-known problem of risk-shifting or asset substitution. A number of studies such as Brennan and Schwartz (1982), Green (1984), Haugen and Senbet (1981, 1987) have argued that convertible/participating debt and other forms of innovations allow these adverse incentives to be mitigated by allowing the financiers to share in any windfall the manager-entrepreneur receives.

The second type of agency problem associated with debt financing is referred to as under-investment problem (Myers, 1977). Here property-owners are motivated to reject positive NPV investment proposals if the wealth enhancement associated with the property mostly accrues to financiers. Bodie and Taggart (1978), Schnabel (1993) and others have argued that innovative vehicles such as participating/convertible debt can be employed to neutralize this problem. Thus, the form of securities issued is crucial in regulating the relationship between the borrower and the financier.

An important reason for the interest in financial system development is that it may influence real decisions. We consider the issue of how the state of development of the financial system can impact the borrower's choice of mortgage financing source and property valuation (see Figure 2). Treatment of financial systems in the existing literature has been somewhat incomplete since its primary goal has been to characterize distributional dynamics. What is missing, specifically, is the variety of financing choices that an investor typically faces in a financial system. Capital markets provide more financing choices to an investor than banks. We study the issue of how the scope of financing available to investors, under different stages of financial system development can effect their decision to invest in properties, an issue that is extremely important to investors, financiers and policy makers. As commercial banks are constrained from investing in property, we demonstrate that in a well developed financial system (capital market dominated); availability of participating mortgages is welfare enhancing. A participating mortgage (PM) is a financial innovation where the lender (capital markets only) accepts below market interest rate in return for a contingent share in the cash flows from operations and/or appreciation in property (Ebrahim, 1996).
We address this vital issue by developing a theoretical model in the context of a specialized banking system (of the U.S. or U.K.) and extend it to that of the universal banking system (of Germany or Japan). In our model, we investigate the impact of financial system development on property valuation in a *rational expectations framework* by modeling the *agency theoretic* perspective of *risk averse* investors (property owners) and financiers (banks/capital markets) under the assumption of increased liquidity as we move from banks to capital markets.\(^{10,11}\) We illustrate that property financing is undertaken in a *pecking order* of increasing *pareto-efficiency* (with reduction in the agency costs of debt) in a three staged process as financial architecture advances from a partially liberalized commercial bank to developed stage of capital markets.

*Three* key results that contribute to the literature on financial system and development and property valuation are derived in this paper. *First*, since commercial banks (in a specialized banking system) are constrained from investing in property, they are generally confined to underwriting plain vanilla (risk-free/risky) mortgages. We illustrate that in the rudimentary stage of a commercial bank, property financing is undertaken in a *pareto-efficient* plain vanilla default-free package that collateralizes the debt in accordance with the prognosis of Scott (1976) and Stulz and Johnson (1985). The bank based equilibria (in general) lies in the lowest rung of *pareto-efficiency*. In general, defaulting debt equilibrium may not be feasible confirming to prognosis of the Myers (1977). However, when the defaulting mortgage equilibrium is feasible, it is at best *pareto-neutral* to default free mortgage equilibrium. Thus, valuation of properties (under pareto-neutral default-free and defaulting debt) constitutes a dilemma for an appraiser as they are contingent on equilibria.

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10 Rational Expectations is defined by Maddock and Carter (1982) as "the application of the principle of rational behaviour to the acquisition and processing of information and to the formation of expectations." Bray (1992) explicates it further by classifying rational expectations equilibrium as "self-fulfilling," as economic agents form correct expectations given the pricing model and information.

11 Diamond (1989) and Hirshleifer and Thakor (1992) have modeled the agency perspective stemming from the conflict of interest between equity (*agent*) versus debt (*principal*). Allen (2001) also encourages banking researchers to adopt this approach.
Figure 2: Impact of Financial System Development on Property Valuation
Second, a pareto-improvement of first solution (commercial bank based equilibria in a specialized system) is obtained by removing the constraint (on ownership of property) on financiers. This moves the equilibria to a more efficient one as it allows universal banks and non-bank financiers (like pension funds, insurance companies) to diversify cross-sectionally in property markets. This interior solution resolves the real estate version of the asset location puzzle as described by Geltner and Miller (2001). However, allowing capital market based financiers like pension funds, insurance companies, etc. (with increased liquidity stemming from financial deepening) yields a decrease in interest rates and an increase in the price of property. This increase in the price of property can also be perceived as ensuing from an increase in its demand by the two competing agents in our economy.

Third, we illustrate a further pareto-enhancement of the above equilibrium under financial innovation by embedding the above default-free mortgage with options (in the form of a Participating Mortgage). This makes the financier's earlier portfolio (of risk-free loan along with fractional purchase of property) redundant yielding a corner solution, where investor owns all property in the economy, while the financier owns a quasi-equity claim of Participating Mortgage. The equilibrium under financial innovation further reduces agency costs embedded in capital market (non-bank) equilibrium in accordance with the prognosis of Green (1984), Haugen and Senbet (1981, 1987) and Schnabel (1993). Participating mortgages are allocatively efficient as the loan-to-value ratio is higher. The reason for this is that the loan includes the price of the option to share in the cash flows from operations and/or appreciation in property. We also indicate the violation of the well-known Black and Scholes (1973) model when applied to pricing contingent claims on property.

Thus, our results differ from the well-known Security Design Irrelevance Theories espoused in Modigliani and Miller (1958), Stiglitz (1974), Baron (1976) and Hellwig (1981). We rationalize our theoretical difference with the above studies due to our methodology of segregating the welfare of the agent and principal of the various (default-free/ defaulting) mortgage contracts under a framework of risk-aversion (i.e., a non-linear valuation scheme),

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12 The asset location puzzle refers to the inconsistency between the theory and the practice of pension fund investment in equities.
where value-additivity espoused in the above theories fails to hold in accordance with the prognosis of Varian (1987).

Do these results have anything to say about how a financial system would evolve if left to its own mechanizations? Our results yield implications for financial system development. *First*, our analysis predicts that an optimal financial system will configure itself skewed towards capital markets irrespective of its origination from a specialized banking system or a universal banking system. A financial system in its infancy will be commercial bank dominated, and increased financial development diminishes bank lending. Greater development in a financial system is manifested in several ways: One is through *financial liberalization*, as there are no restrictions on LTV ratio and financiers are not prohibited from direct ownership of property. Thus, capital markets develop as there are restrictions on banks activities. Hence, we resolve the issue of how regulatory constraints, aimed principally at banking scope affect the evolution of financial system. Our results are in conformity with the empirical findings of Cho (1988), Chari & Henry (2003) and Demetriades *et al* (2001). Another manifestation is through *financial deepening*, which enhances loan-to-value ratio of the mortgages and thus the *allocative* efficiency of capital.\(^\text{13}\) Our results are in conformity with the empirical findings of King and Levine (1993) and Levine *et al* (2000). A further manifestation is through *financial innovation* that reduces endogenous agency costs of debt contrary to the findings of Modigliani and Miller (1958), Stiglitz (1974), Baron (1976) and Hellwig (1981) as stated earlier. Hence, the welfare relevance of capital markets should grow through time as financial system develops. Our analysis is in conformity with that of Scharfstein (1988), Weinstein and Yafeh (1998) and Allen and Gale (2000). However, it differs with that of Rajan and Zingales (1999) and Tadesse (2001), who provide a strong case for bank financing.

*Second*, we also rationalize the co-existence of banks and financial markets in a well-developed financial system (irrespective of the type of system). While our model demonstrates that a borrower chooses either bank financing or capital market financing, we could envision financing options lying along a continuum ranging from plain vanilla

\(^{13}\) For more details, see Ebrahim and Mathur (2000).
defaulting mortgage (by banks) to default-free participating mortgage (by capital market financiers) as the polar extremes. The key determinants of financing choices are constituted by the quality of the underlying collateral (described by the return-risk distribution of the property) and the risk preferences of the agents in the economy. The general results described above (of the pareto-advancement of equilibria) are valid for collateral (which holds it value in the future) and high risk aversion of agents. However, for investors of low quality collateral (such as mobile homes), where the terminal payoffs are zero for some state of the economy, defaulting mortgage financing (if feasible) may be the only option. Furthermore, agents with low risk aversion may be indifferent to bank and capital market financing as demonstrated in our trivial solutions.

This paper is organized as follows: Section II illustrates the theoretical underpinnings of the model, while Section III evaluates the model solutions with key results. Finally, Section IV provides the concluding remarks.

**II. MODEL DEVELOPMENT**

For simplicity and mathematical tractability, we assume a two period economy, where there are two types of agents (namely, an investor and a financier) who maximize their respective welfare. There are two types of assets in this economy. One is a real asset (i.e., property) with Net Operating Income (NOI) ($q_1$) and a liquidating value ($P_1$), where $q_1$ and $P_1$ are positive random first-order Markov processes. The other is a financial asset (i.e., mortgage - default-free/ defaulting loan) created by trading off claims against the payoffs of the real asset (see Figure 3). The financial assets range from plain vanilla loans (mortgages) in the rudimentary stage of a bank to participating ones in the developed stage of capital markets. Different financiers are considered ranging from a partially liberalized commercial banks (in a specialized banking system – with no restrictions on loan-to-value ratio but barred from direct ownership of property) to capital markets. This encapsulates the dimensions of financial development (see Figure 4). Finally, the analysis is carried out by modeling the objective functions of the investor and the financier, imposing the market clearing conditions for all the loans and solving for optimal asset and mortgage pricing components.
Figure 3: Various Mortgage Financing Options for an Investor

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial Liberalization</strong></td>
<td>Loan-to-Value (LTV) Ratio. In our model, LTV Ratio increases as financing advances from rudimentary stage of partially liberalized commercial banks (with restrictions on direct ownership in property) to developed stage of liberalized capital markets (with no restrictions on direct ownership in property).</td>
</tr>
<tr>
<td><strong>Financial Deepening</strong></td>
<td>Increased supply of funds (credit) in a developed financial system (capital market dominated). In our model, the endowments of financier increases as financing advances from rudimentary stage of commercial bank to developed stage of capital markets.</td>
</tr>
<tr>
<td><strong>Financial Innovation</strong></td>
<td>Quasi-Equity (Participating) Mortgages. Availability of participating mortgages as financing advances from rudimentary stage of commercial bank to developed stage of capital markets.</td>
</tr>
</tbody>
</table>

Figure 4: Model Parameters for Financial Innovation, Financial Liberalization and Financial Deepening.

II.a. Modeling the Objective Function of the Investor

The goal of the investor is to optimize the expected utility of consumption

\[
Max. \ E_0 \{U(c_0) + \gamma U(c_1)\}
\]

\((in \ Q, c_0, c_1, s)\)

subject to the temporal budget constraints

\[c_0 = e_0 + Q - sP_0\]  \hspace{1cm} (1)

\[c_1 = e_1 + s(\tilde{q}_1 + \tilde{P}_1) - Q[1 + \tilde{r} + \Phi\text{Max}[0, (\tilde{q}_1 + \tilde{P}_1 - P_0)]]\]  \hspace{1cm} (2)

where \(E_0 \{\cdot\}\) is the expectation operator at time 0, \(U(\cdot)\) is the concave and differentiable Von Neumann-Morgenstern utility function of investor, \(\gamma\) is the discount factor, \(s\) is the fractional
investment in the property, \( Q \) is the amount of capital borrowed, \( P_0 \) is the price of the property at time 0 (incorporating all relevant transaction costs), \( e_0 \) is the endowment at time 0, \( \tilde{r} \) is the real interest rate, \( \tilde{q}_1 \) is the NOI of the property at time 1, \( \tilde{P}_1 \) is the liquidating value of the property at time 1, \( c_0 \) is the consumption of the investor at time 0, \( \tilde{c}_1 \) is the consumption of the investor at time 1, \((1-k_{nc})\%\) is the sum of the direct and indirect bankruptcy costs, \( \Phi \) is the participating rate on the NOI plus the appreciation of the property per dollar of the loan at time 1, the notations \( Z, P, DF \) and \( D \) denote the terms 'critical state of the economy', 'participating', 'default-free' and 'defaulting' respectively.

The budget constraint at \( t = 0 \) (Equation 1) illustrates consumption utilizing the initial endowment \( (e_0) \) after deducting \( sP_0 \) for the purchase of \( s \) fraction of a property financed by a mortgage (loan) of \( Q \). The budget constraint at \( t = 1 \) (Equation 2) incorporates consumption from the future endowment \( (e_1) \) in addition to the payoffs of \( s \) fraction of a property after deducting the mortgage payment with interest and contingent participation in property payoffs \( \{s(\tilde{q}_1+\tilde{P}_1) - Q[1+\tilde{r}+\Phi\text{Max}[0, (\tilde{q}_1 + \tilde{P}_1 - P_0)]\} \).\(^{14}\) Thus, non-property resources (ensuing from initial endowment and loan proceeds) are expended in period zero to consume property related payoffs (net of mortgage payment) in period one.

The Lagrangian \( L \) can be written as:

\[
L = E_0\{[U(c_0)+\gamma U(\tilde{c}_1)] + \lambda_0[e_0 - sP_0+Q - c_0] \\
+ \lambda_1\gamma [e_1+ [s (\tilde{q}_1 + \tilde{P}_1) - Q[1 + \tilde{r}+\Phi \text{ Max } [0, (\tilde{q}_1 + \tilde{P}_1 - P_0)]\] - \tilde{c}_1]\}
\]

The First Order Necessary Conditions (FONCs or Euler Equations) are given by:

(i) At the margin, the benefit of borrowing is equal to its associated cost. This simplifies to the demand function for a mortgage described as follows. The intertemporal marginal rate of substitution (IMRS) of investor \([\text{IMRS}_1 = \gamma(U(\tilde{c}_1)/U'(c_0))\] times the compound factor, consisting of one plus the real rate of interest plus the contingent participation in property payoffs, is equal to the unit value of resources loaned.

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\(^{14}\) The net payoffs of a property in the non-default states of the economy consist of the following components:

(a) The inflow component stemming from fractional ownership of the property times the NOI added to the liquidating value.

(b) The outflow component stemming from mortgage payment.
\[ \gamma \mathbb{E}_0 \left\{ \left[ \frac{U'(\tilde{c}_{1i})}{U'(c_0)} \right] \left[ 1 + r + \Phi \text{Max} \left[ 0, (q_{1i} + \tilde{P}_{1i} - P_0) \right] \right] \right\} = 1 \tag{3} \]

For Plain Vanilla Risk-free (RF or Default-free) Mortgage, Equation (3) simplifies to

\[ \gamma \mathbb{E}_0 \left\{ \left[ \frac{U'(\tilde{c}_{1i})}{U'(c_0)} \right] \left[ 1 + r_{RF} \right] \right\} = 1 \tag{3a1} \]

For Default-free Participating Mortgage (DFPM), Equation (3) simplifies to\(^{15}\)

\[ \gamma \int_0^{Z_p} \left[ \frac{U'(\tilde{c}_{1ij})}{U'(c_0)} \right] \left[ 1 + r_{DFPM} \right] dx + \gamma \int_{Z_p}^{\infty} \left[ \frac{U'(\tilde{c}_{1ij})}{U'(c_0)} \right] \left[ 1 + r_{DFPM} + \Phi \left( q_{1ij} + \tilde{P}_{1ij} - P_0 \right) \right] dx = 1 \tag{3a2} \]

Equation (3a2) is derived by decomposing the expectations operator in Equation (3) into two integral components: The first integral incorporates the poor states of the economy (below the critical state 'Z\(_p\)'), while the second integral incorporates the good states of the economy (above the critical state 'Z\(_p\)' – See Figure 5 below).

Figure 5: Payoff diagram for Default-free Mortgage Contracts

For Plain Vanilla Risky (Defaulting) Mortgage, Equation (3) simplifies to\(^{16}\)

\(^{15}\) The variable of integration \(x\) is defined in terms of the probability density function of terminal payoffs (\(f(\cdot)\)) as follows: \(dx = f(q_{1ij} + P_{1ij})d(q_{1ij} + P_{1ij})\).

\(^{16}\) The critical state of default 'Z' is defined as the future state of the economy up to which the investor is
Equation (3b1) is derived by decomposing the expectations operator in Equation (3) into two integral components: The first integral incorporates default states of the economy (below the critical state ‘Z’), while the second integral incorporates the normal states of the economy (above the critical state ‘Z’ – See Figure 6 below).

Figure 6: Payoff diagram for Defaulting Mortgage Contracts

For Defaulting Participating Mortgage (DPM), Equation (3) simplifies to

\[
\begin{align*}
\gamma \int_{0}^{Z} \left[ \frac{U'(c_{ij})}{U'(c_{0})} \right] \left[ q_{ij} + P_{ij} \right] dx + \gamma \int_{Z}^{\infty} \left[ \frac{U'(c_{ij})}{U'(c_{0})} \right] \left[ 1 + r_{DPM} \right] dx + \\
+ \gamma \int_{Z p_2}^{\infty} \left[ \frac{U'(c_{ij})}{U'(c_{0})} \right] \left[ 1 + r_{DPM} + \Phi \left( q_{ij} + P_{ij} - P_0 \right) \right] dx = 1
\end{align*}
\] (3b2)

technically in default of his mortgage obligations.
That is, \( Q_{\text{Risky}}^* (1 + r_{\text{Risky}}^*) > (q_{ij} + P_{ij}) \forall j \leq Z. \)

Likewise, the critical state of default ‘Z_{p1}’ is defined as the future state of the economy up to which the investor is technically in default of his mortgage obligations. That is, \( Q_{\text{DPM}}^* (1 + r_{\text{DPM}}^*) > (q_{ij} + P_{ij}) \forall j \leq Z_{p1}. \)
Equation (3b2) is derived by decomposing the expectations operator in Equation (3) into three integral components: The first integral incorporates the default states of the economy (below the critical state ‘Zp1’), while the second integral incorporates the poor states of the economy (between critical states ‘Zp1’ and ‘Zp2’) and the third integral incorporates the good states of the economy (above critical state ‘Zp2’) of the economy.

(ii) At the margin, the investor will only bid for that fraction of a property, which makes the net benefit of ownership equal to zero. Similarly, the investor will avoid investing in a property if net benefits are less than zero. This simplifies to the demand function for a property described as follows. The price of the property bid by a prospective owner is equal to the IMRS of the investor (IMRSi) times the proceeds from the NOI plus the liquidating value.

\[
P_0 = \gamma E_0 \left\{ \frac{U'(c_{ij})}{U'(c_{0})} \right\} \left[ q_{ij} + \tilde{P}_{ij} \right] = \gamma \int_0^\infty \frac{U'(c_{ij})}{U'(c_{0})} \left[ q_{ij} + \tilde{P}_{ij} \right] dx
\]

\[
= \gamma \int_0^Z \frac{U'(c_{ij})}{U'(c_{0})} \left[ q_{ij} + \tilde{P}_{ij} \right] dx + \gamma \int_Z^\infty \frac{U'(c_{ij})}{U'(c_{0})} \left[ q_{ij} + \tilde{P}_{ij} \right] dx
\]

(4)

For Default-Free (Plain-Vanilla and Participating) Mortgages, \(P_0\) is described by Equation (4)

However, for the Plain Vanilla Risky (Defaulting) Mortgage, Equation (4) simplifies to

\[
P_0 = \gamma \int_Z^\infty \frac{U'(c_{ij})}{U'(c_{0})} \left[ q_{ij} + \tilde{P}_{ij} \right] dx \quad (4a1)
\]

Equation (4a1) is derived by decomposing the expectations operator in Equation (4) into two integral components. The first integral has zero value as it incorporates default states of the economy (below the critical state ‘Z’), while the second integral incorporates the normal states of the economy (above the critical state ‘Z’). It should be
noted that in the context of rational expectations, defaulting mortgages are *value-reducing*.

Furthermore, for the Defaulting Participating Mortgage, Equation (4) simplifies to

\[ P_0 = \gamma \int_{Z_{pl}}^{\infty} \left[ \frac{U'(c_{1j})}{U'(c_0)} \right] \left[ q_{ij} + \bar{P}_{ij} \right] dx \]  

Equation (4a2) is derived by decomposing the expectations operator in Equation (4) into two integral components. Here too, the first integral has zero value as it incorporates default states of the economy (below the critical state 'Z_{pl}') while the second integral incorporates the normal states of the economy (above the critical state 'Z_{pl}'). Here too, the defaulting mortgages are *value-reducing*.

The above equations are similar to the two-period version of the consumption capital asset pricing model (CCAPM) (Breeden, 1979) incorporating all relevant features of the property market.

Thus, maximization of the investor's objective requires that the following conditions are met: The deterministic budget constraint (at t = 0) in Equation (1), and the stochastic budget constraint (for each state of the economy at t = 1), as shown by Equation (2), are satisfied; The simplified versions of FONCs represented by Equations (3) and (4) are satisfied; The second order conditions for a maximum are satisfied. We note that it can be demonstrated that maximization of a concave and differentiable objective (utility) function with linear constraints gives a negative definite bordered Hessian matrix.

**II.b. Modeling the Objective Function of the Financier**

Similar to the previous case, the goal of the financier is to maximize her two-period utility of consumption.

Max. \( E_0 \{ V(c_0') + \gamma V(c_1') \} \)

\( (\text{in } Q', c_0', c_1', s') \)
subject to the temporal budget constraints
\[ c'_{0} = e'_{0} - s'P_{0} - Q' \]  
\[ c'_{1} = e'_{1} + s'[\tilde{q}_{1} + \tilde{P}_{1}] + Q'[1 + \tilde{r} + \Phi \max[0, (\tilde{q}_{1} + \tilde{P}_{1} - P_{0})]] \]

where \( V(\cdot) \) is the concave and differentiable Von Neumann-Morgenstern utility function of the financier, while the notation with primes have the same meaning as that for the investor.

The budget constraint at \( t = 0 \) (Equation 5) denotes consumption stemming from the initial endowment \( (e'_{0}) \) after purchasing a fraction \( (s') \) of property and disbursing a mortgage of \( Q' \). The budget constraint at \( t = 1 \) (Equation 6) represents consumption resulting from the future endowment \( (e'_{1}) \) along with the reimbursement of the mortgage payment with interest and contingent participation in property payoffs \( Q'[1 + \tilde{r} + \Phi \max[0, (\tilde{q}_{1} + \tilde{P}_{1} - P_{0})]] \). Here too, non-property resources (emanating from the initial endowment) are expended in period zero to consume property related payoffs in period one.

The Lagrangian \( L' \) can be written as:
\[ L' = E_{0}\{[V(c'_{0}) + \gamma' V(c'_{1})] + \lambda'_{0}[e'_{0} - s'P_{0} - Q' - c'_{0}] + \lambda'_{1} \gamma' [e'_{1} + s'[\tilde{q}_{1} + \tilde{P}_{1}] + Q'[1 + \tilde{r} + \Phi \max[0, (\tilde{q}_{1} + \tilde{P}_{1} - P_{0})] - c'_{1}]\} \]

The First Order Necessary Conditions (FONCs or Euler Equations) are given by:

(i) At the margin, the benefit of lending should equal its associated cost. This simplifies to the supply function for a mortgage described as follows. The IMRS of the financier (IMRS\(_{F}\)) times the compound factor, consisting of one plus the real rate of interest plus contingent participation in property payoffs is equal to the unit value of the resources loaned.
\[ \gamma' E_{0}\{[V(c'_{1})] [1 + \tilde{r} + \Phi \max[0, (\tilde{q}_{1} + \tilde{P}_{1} - P_{0})]]\} = 1 \]  
\[ (7) \]

For Plain Vanilla Risk-free (Default-free) mortgage, equation (7) simplifies to
\[ \gamma' E_{0}\{[V(c'_{1})] [1 + r_{RF}]\} = 1 \]
\[ (7a1) \]
For Default-free Participating Mortgage, equation (7) simplifies to

\[ \gamma' \int_{0}^{Z_p} \frac{V'(c'_{1j})}{V'(c'_{0})} \left[ 1 + r_{DFPM} \right] dx + \gamma' \int_{Z_p}^{\infty} \frac{V'(c'_{1j})}{V'(c'_{0})} \left[ 1 + r_{DFPM} + \Phi (q_{ij} + \tilde{P}_{ij} - P_0) \right] dx = 1 \tag{7a2} \]

Equation (7a2) is derived by decomposing the expectation operator in Equation (7) into two integral components. The first integral incorporates poor states of the economy (below the critical state 'Zp'), while the second integral incorporates good states of the economy (above the critical state 'Zp' – See again Figure 5).

For Plain Vanilla Risky (Defaulting) Mortgage, equation (7) simplifies to

\[ \gamma' k_{BC} \int_{0}^{Z} \frac{V'(c'_{1j})}{V'(c'_{0})} \left[ \frac{q_{ij} + \tilde{P}_{ij}}{Q'_{Risky}} \right] dx + \gamma' \int_{Z}^{\infty} \frac{V'(c'_{1j})}{V'(c'_{0})} \left[ 1 + r_{Risky} \right] dx = 1 \tag{7b1} \]

Equation (7b1) is derived by decomposing the expectation operator in Equation (7) into two integral components. The first integral incorporates default states of the economy (below the critical state 'Z'), while the second integral incorporates normal states of the economy (above the critical state 'Z'). The first integral reflects the fact that, in bankruptcy, the financier receives proceeds net of all direct and indirect costs of bankruptcy denoted by the fraction \((1-k_{BC})\). In contrast, the second integral reflects full contractual payments (of principal, interest and contingent participation in property payoffs) in the normal states of the economy.

For Defaulting Participating Mortgage, equation (7) simplifies to

\[ \gamma' k_{BC} \int_{0}^{Z_{p1}} \frac{V'(c'_{1j})}{V'(c'_{0})} \left[ \frac{q_{ij} + \tilde{P}_{ij}}{Q'_{DPM}} \right] dx + \gamma' \int_{Z_{p1}}^{Z_{p2}} \frac{V'(c'_{1j})}{V'(c'_{0})} \left[ 1 + r_{DPM} \right] dx + \gamma' \int_{Z_{p2}}^{\infty} \frac{V'(c'_{1j})}{V'(c'_{0})} \left[ 1 + r_{DPM} + \Phi (q_{ij} + \tilde{P}_{ij} - P_0) \right] dx = 1 \tag{7b2} \]
Equation (7b2) is derived by decomposing the expectations operator in Equation (7) into three integral components: The first integral incorporates the default states of the economy (below the critical state 'Z_{p1}'), while the second integral incorporates the poor states of the economy (between critical states 'Z_{p1}' and 'Z_{p2}') and the third integral incorporates the good states of the economy (above critical state 'Z_{p2}') of the economy. Here too, the first integral reflects the fact that, in bankruptcy, the financier receives proceeds net of all direct and indirect costs of bankruptcy denoted by the fraction \((1-k_{BC})\). In contrast, the last two integrals reflect full contractual payments in the normal states of the economy.

(ii) At the margin, the financier will only bid for that fraction of a property, which makes her net benefit of ownership equal to zero. Similarly, the financier will avoid investing in a property if her net benefits are less than zero. This simplifies to the demand function for a property described as follows. The price of the property bid by a financier is equal to the IMRS of the financier (IMRS_{f}) times the proceeds from the NOI plus the liquidating value.

\[
P_0 = \gamma' E \left\{ \frac{V'(c'_{1j})}{V'(c'_{0j})} \left[ q_1 + P_{1j} \right] \right\} = \gamma' \int_{0}^{\infty} \left[ \frac{V'(c'_{1j})}{V'(c'_{0j})} \right] \left[ q_1 + P_{1j} \right] dx \quad (8)
\]

For Default-free (Plain-Vanilla and Participating) mortgages, \(P_0\) is described by Equation (8).

(iib1) However, for the Plain Vanilla Risky (Defaulting) Mortgage, Equation (8) simplifies to

\[
P_0 = \gamma' \int_{Z}^{\infty} \left[ \frac{V'(c'_{1j})}{V'(c'_{0j})} \right] \left[ q_1 + P_{1j} \right] dx \quad (8a1)
\]

Equation (8a1) is derived by decomposing the expectation operator in Equation (8) into two integral components: The first integral in the default states of the economy (below
the critical state \('Z'\) has a value of zero. In contrast, the second integral incorporates the normal states of the economy (above the critical state \('Z'\)). Here too, defaulting mortgages are value reducing.

Furthermore, for the Defaulting Participating Mortgage, Equation (8) simplifies to

\[
P_0 = \gamma' \int_{Z_{p1}}^{\infty} \frac{V'(c_{1j}^{'})}{V'(c_{0}^{'})} [q_{ij} \tilde{P}_{ij} + \tilde{P}_{ij}] \, dx \quad (8a2)
\]

Equation (8a2) is derived by decomposing the expectation operator in Equation (8) into two integral components: The first integral in the default states of the economy (below the critical state \('Z_{p1}'\)) has a value of zero. In contrast, the second integral incorporates the normal states of the economy (above the critical state \('Z_{p1}'\)). Here too, defaulting mortgages are value reducing.

Thus, maximization of the financier's objective requires that the following conditions are satisfied: The deterministic budget constraints in both periods represented by Equations (5) and (6) are satisfied; the simplified FONCs, i.e., Equation (7) and (8) are satisfied; the second order condition for a maximum is satisfied, based on our previous argument for a concave and differentiable utility function with linear constraints.

II.c. Market Clearing Conditions and Regulatory Constraints

The following conditions are necessary for equilibrium:

(i) For the mortgage market to be in equilibrium:

\[
\text{Funds Borrowed (Q) = Funds Lent (Q')} \quad (9)
\]

(ii) In specialized banking system (such as that of the U.S. and U.K.), banks are barred from direct ownership of property due to Central Bank (Federal Reserve) regulations (Greenbaum and Thakor, 1995). For the asset (property) market to be in equilibrium, this implies that: \(s = 1\) \quad (10a)

However, non-bank financiers (and capital market financiers) can partake in direct investment of property. For the asset (property) market to be in equilibrium, this
implies that: $s + s' = 1$. Finally, since one cannot sell short a property: $s \geq 0$ and $s' \geq 0$.

(10b)

III. MODEL SOLUTIONS

A Rational Expectations Equilibrium (REE) is defined as the one where all agents in the economy are knowledgeable of the NOI from the property ($\widetilde{q}_t$), the liquidating value of the property at time ($\widetilde{P}_t$) and their related probability distributions. Assuming competitive markets and no initial capital constraints, at most four distinct solutions are feasible for risk-averse investor under default-free and defaulting mortgage contracts upon satisfaction of necessary conditions (discussed below). The default-free mortgages involve at most two distinct solutions (that of risk-free and participating mortgage as illustrated in Figure 5), while the defaulting mortgages involve at most the remaining two (that of risky and participating mortgage as illustrated in Figure 6). The equilibria are ranked in pecking order of increasing pareto-efficiency attributed to minimization of endogenous agency costs of mortgage. If the investor owns the property in its entirety (referred to in the finance literature as the asset allocation puzzle, i.e., $s = 1$), the solution is termed as a corner solution in contrast to the interior solution ($s \in (0, 1)$), where both the agents own a fraction of the property.

III.a. Necessary Conditions for Model Solutions

Proposition 1: The Default Free Mortgage equilibria require satisfaction of the three conditions given below.

(i) Basic Condition: The terminal payoffs of a property (composed of the sum of NOI plus the liquidating value) is strictly positive even in the worst state of the economy (in the following period). That is Min. ($\widetilde{q}_t + \widetilde{P}_t$) $> 0$, $\forall j$.

(ii) Mortgage Pricing Condition: This requires the equality between demand and supply functions for mortgage financing.

(iia) For Plain Vanilla Risk-free (Default-free) Mortgage:
\( \gamma E_0 \left\{ \frac{U'(\tilde{c}_1)}{U'(c_0)} \left[ 1 + r_{RF} \right] \right\} = \gamma' E_0 \left\{ \frac{V'(\tilde{c}')}{V'(c'_0)} \left[ 1 + r_{RF} \right] \right\} = 1 \) \hfill (11a)

(ii) For Default-free Participating Mortgage

\[
\gamma \int_0^{Z_p} \left[ \frac{U'(\tilde{c}_{1j})}{U'(c_0)} \right] \left[ 1 + r_{DFPM} \right] dx + \gamma \int_{Z_p}^{\infty} \left[ \frac{U'(\tilde{c}_{1j})}{U'(c_0)} \right] \left[ 1 + r_{DFPM} + \Phi (q_{ij} + \tilde{p}_{ij} - P_0) \right] dx = 1 = \\
\gamma' \int_0^{Z_p} \left[ \frac{V'(\tilde{c}'_{1j})}{V'(c'_0)} \right] \left[ 1 + r_{DFPM} + \Phi (q_{ij} + \tilde{p}_{ij} - P_0) \right] dx \hfill (11b)
\]

The above equations imply that for equilibrium to exist, the IMRS of both agents in the economy must adjust to solve for the unique price of the loan in terms of the interest/participation rate and the loan amount. This result indicates that the market for a mortgage is not monopolistic as neither agent has the market power to wrest any economic surplus from the other.

(iii) Asset (Property) Pricing Condition: This requires the price of property to equal the expected value of the IMRS of the investor (IMRS\(_I\)) times the net proceeds of the property (after repayment of the mortgage amount) in the following period. This is further equal to the IMRS of financier (IMRS\(_F\)) times the net proceeds of the property (assuming \( s' \neq 0 \)).

\[ P_0 = \gamma E_0 \left\{ \left[ \frac{U'(\tilde{c}_1)}{U'(c_0)} \right] [\tilde{q}_1 + \tilde{p}_1] \right\} = \gamma' E_0 \left\{ \left[ \frac{V'(\tilde{c}_')}{V'(c'_0)} \right] [\tilde{q}_1 + \tilde{p}_1] \right\}, \quad \forall s' \neq 0 \] \hfill (12a)

\[ P_0 = \gamma E_0 \left\{ \left[ \frac{U'(\tilde{c}_1)}{U'(c_0)} \right] [\tilde{q}_1 + \tilde{p}_1] \right\} \forall s' = 0 \] \hfill (12b)

**Proof:**

(i) This condition is attributed to the fact that a property (along with the underlying land) serves as collateral behind the loan. Since the land and the attached structure is able to retain some value (in the following period), it enables the investor (property owner) to
pay back the default-free mortgage with contractual amount in all states of the economy.

(ii) (iia) Equation (11a) is derived from Equations (3a1), (7a1) and (9).

(iib) Equation (11b) is derived from Equations (3a2), (7a2) and (9).

(iii) (iiia) Equation (12a) is derived from Equations (4), (8) and (10b).

(iiiib) Equation (12b) is derived from Equations (4) and (10a). 

Q.E.D.

**Proposition 2:** The Defaulting Mortgage equilibria require satisfaction of the three conditions given below.

(i) **Basic Conditions:**
   (a) The mortgage is structured in such a way that it involves a default in some state of the economy in the following period;
   (b) The interest rate contracted for defaulting mortgage is greater than that of the default-free mortgage solution determined above;
   (c) The loan-to-value (debt) ratio for defaulting mortgage is greater than that of the default-free mortgage solution determined above.

(ii) **Mortgage Pricing Condition:** This requires the equality between demand and supply functions for mortgage financing.

(iiia) For Plain Vanilla Risky (Defaulting) Mortgage

\[
\gamma \int_0^Z \left[ \frac{U'(c_{1\tilde i})}{U'(c_0)} \right] \left[ \frac{q_{1\tilde i} + P_{1\tilde i}}{Q_{\text{Risky}}} \right] dx + \gamma \int_Z^\infty \left[ \frac{U'(c_{1\tilde i})}{U'(c_0)} \right] [1 + r_{\text{Risky}}] dx = 1 = \\
\gamma' k_{\text{IC}} \int_0^Z \left[ \frac{V'(c'_{1\tilde i})}{V'(c'_{0})} \right] \left[ \frac{q_{1\tilde i} + P_{1\tilde i}}{Q_{\text{Risky}}} \right] dx + \gamma' \int_Z^\infty \left[ \frac{V'(c'_{1\tilde i})}{V'(c'_{0})} \right] [1 + r_{\text{Risky}}] dx \quad (13a)
\]

(iiib) For Defaulting Participating Mortgage
\[ \gamma \int_{0}^{Z_{p1}} \left[ \frac{U'(c_{1j})}{U'(c_0)} \right] \left[ q_{1j} + \frac{\tilde{P}_{1j}}{Q_{DPM}} \right] dx + \gamma \int_{Z_{p1}}^{Z_{p2}} \left[ \frac{U'(c_{1j})}{U'(c_0)} \right] \left[ 1 + r_{DPM} \right] dx + \]

\[ + \gamma \int_{Z_{p2}}^{\infty} \left[ \frac{U'(c_{1j})}{U'(c_0)} \right] \left[ 1 + r_{DPM} + \Phi (q_{1j} + \tilde{P}_{1j} - P_0) \right] dx = 1 = \gamma \int_{0}^{Z_{p1}} \left[ \frac{V'(c'_{1j})}{V'(c'_{0})} \right] \left[ q_{1j} + \frac{\tilde{P}_{1j}}{Q'_{DPM}} \right] dx + \gamma \int_{Z_{p1}}^{Z_{p2}} \left[ \frac{V'(c'_{1j})}{V'(c'_{0})} \right] \left[ 1 + r_{DPM} \right] dx + \]

\[ + \gamma \int_{Z_{p2}}^{\infty} \left[ \frac{V'(c'_{1j})}{V'(c'_{0})} \right] \left[ 1 + r_{DPM} + \Phi (q_{1j} + \tilde{P}_{1j} - P_0) \right] dx \quad (13b) \]

(iii) **Asset (Property) Pricing Condition:** This requires the price of property to equal the expected value of the IMRS of the investor (property owner) (IMRS\(_I\)) times the net proceeds of the property (after repayment of the mortgage amount) in the following period. This is further equal to the IMRS of financier (IMRS\(_F\)) times the net proceeds of the property (assuming \(s' \neq 0\)).

(iii-a) For Plain Vanilla Risky (Defaulting) Mortgage

\[ P_0 = \gamma \int_{0}^{Z} \left[ \frac{U'(c_{1j})}{U'(c_0)} \right] \left[ q_{1j} + \tilde{P}_{1j} \right] dx = \gamma \int_{Z}^{\infty} \left[ \frac{V'(c'_{1j})}{V'(c'_{0})} \right] \left[ q_{1j} + \tilde{P}_{1j} \right] dx, \ \forall \ s' \neq 0 \quad (14a) \]

\[ P_0 = \gamma \int_{0}^{Z} \left[ \frac{U'(c_{1j})}{U'(c_0)} \right] \left[ q_{1j} + \tilde{P}_{1j} \right] dx \ \forall \ s' = 0 \quad (14b) \]

(iii-b) For Defaulting Participating Mortgage

\[ \text{The quasi-equity nature of defaulting mortgages ensures that } s' = 0 \text{ in both risky mortgages as well as participating mortgages. In other words, the price of property under defaulting mortgages is given by Equations (14b) and (14d) respectively.} \]
\[ P_0 = \gamma \int_{\mathcal{Z}_{p1}}^{\infty} \left( \frac{U'(c_i)}{U'(c_0)} \right) \left[ q_{ij} + \widetilde{p}_{ij} \right] dx = \gamma' \int_{\mathcal{Z}_{p1}}^{\infty} \left( \frac{V'(c'_i)}{V'(c'_0)} \right) \left[ q_{ij} + \widetilde{p}_{ij} \right] dx, \quad \forall \ s' \neq 0 \] (14c)

\[ P_0 = \gamma \int_{\mathcal{Z}_{p1}}^{\infty} \left( \frac{U'(c_i)}{U'(c_0)} \right) \left[ q_{ij} + \widetilde{p}_{ij} \right] dx \quad \forall \ s' = 0 \] (14d)

**Proof:**

(i)  (a) For a mortgage to be Risky or Participating with default, the borrower (investor) fails to honor his contractual obligations in some state of the economy in the future. This implies that: \([q_{ij} + \widetilde{p}_{ij}] - Q^*_{\text{Risky/DPM}} (1 + r_{\text{Risky/DPM}}) < 0\) for some state of the economy in period 1;

(b & c) The reason why the loan-to-value (debt) ratio and the contract rate of interest are higher than that of risk-free mortgage is due to the fact that the supply curve is upward sloping. The borrower prefers high mortgage (debt) ratio, while the financier seeks extra compensation for it and for states of default.

(ii) (iia) Equation (13a) is derived from Equations (3b1), (7b1) and (9).

(iib) Equation (13b) is derived from Equations (3b2), (7b2) and (9).

(iii) (iiia) Equation (14a) is derived from Equations (4a1), (8a1) and (10b).

(iiib) Equation (14b) is derived from Equations (4a1) and (10a).

(iiic) Equation (14c) is derived from Equations (4a2), (8a2) and (10b).

(iiid) Equation (14d) is derived from Equations (4a2) and (10a). \(\text{Q.E.D.}\)

**III.b. Key Results**

**Theorem 1**

Property financing is undertaken in a pecking order of increasing **pareto-efficiency** (that minimizes the **endogenous** agency costs of a mortgage) as it advances from partially liberalized banks to capital markets. This increase in the efficiency of financing is accompanied by an improvement in the value of the underlying collateral, i.e., property price \((P_0)\).
The following general results can be inferred from the model: First, the primary solution is obtained in the rudimentary stage of commercial banks, where the default-free mortgages are pareto-optimal to defaulting mortgages in accordance with the prognosis of Scott (1976) and Stulz and Johnson (1985). Second, a pareto-improvement of the first solution is obtained in capital markets, where non-bank financiers like pension funds, insurance companies, etc., own a fraction of equity position in the property. This solution resolves the real estate version of the asset location puzzle (see Geltner and Miller, 2001). Third, a further pareto-enhancement of the second equilibrium is obtained with financial innovation by embedding the above default-free mortgage with options (in the form of a participating mortgage) in accordance with the prognosis of Green (1984) and Haugen and Senbet (1981, 1987) (See Figures 1 and 7).

Proof:

The four equilibria described in Propositions 1 and 2 are derived using optimization techniques and thus constitute optimal financing packages. However, the equilibria are impacted differentially by the endogenous agency costs of debt, which are sequentially reduced as financing advances from (i) banks to capital markets and (ii) plain vanilla debt to participating debt as explained below:

(ia) Commercial Bank Based Equilibria: The Lowest Rung of Pareto-Efficiency.

Since commercial banks are constrained from investing in property $s' = 0$ as indicated in Equation (10a), they are confined to underwriting plain vanilla (risk-free/ risky) mortgages. Here we derive a general result of pareto-optimality of default-free mortgages over defaulting mortgages as explained below. This result holds true as we move across financiers (i.e., from banks to capital markets) and across financial products (i.e., plain vanilla debt to participating debt).

The defaulting mortgage pricing conditions (see Figure 6 and Equations (13a) and (13b)) consist of integrals incorporating default states (below the critical state '$Z/Z_p$') and the normal states (at or above the critical state '$Z/Z_p$') of the economy. In the default state, the financier recoups her capital by repossessing the property and benefits
from the liquidation value of the property. But due to the direct and indirect costs of bankruptcy, she receives a fraction \( k_{BC} \) of the residual value of the property in contrast to the normal states of the economy, where she receives the full contractual payments of the mortgage. Thus, in equilibrium, the bankruptcy costs are incorporated in such a way that the financier does not face them. It is the investor (property owner) who bears the costs in the form of higher interest rates.

Since a property and the underlying land retain some value in the future, it implies the satisfaction of the Basic Condition of Proposition 1. That is, \( \min (q_{ij} + P_{ij}) > 0, \forall j \). Here, the defaulting mortgages are subject to bankruptcy costs. Since bankruptcy costs are conveyed to the investor (as explained above) their welfare is lower with defaulting mortgages than with default-free mortgages. Furthermore, equilibria with defaulting mortgages are feasible only when the bankruptcy costs are capped or under certain limits. In contrast, equilibria with default-free mortgages are feasible even when those with defaulting mortgages are uneable due to excessive bankruptcy costs. In this case, defaulting facilities (if they exist) are at best pareto-neutral to their default-free counterparts. This result is consistent with finance literature as Myers (1977) attributes agency cost of mortgage restraining firms from investing in positive NPV projects leading to underinvestment issue. One way to alleviate this agency cost of mortgage is to collateralize mortgage with tangible assets of a firm as discussed in Stulz and Johnson (1985). This is precisely what our model entails in our design of the default-free mortgage facilities by ensuring that investors pledge the underlying property as collateral to financiers as implied by the Basic Condition of Proposition 1.\(^{19}\)

(ib) The property appraisal dilemma under pareto-neutral equilibria and the optimization of their values under pareto-dominant default-free mortgage equilibrium.

\(^{19}\) It should be noted that results opposite to the above general one is obtained when \( \min (q_{ij} + P_{ij}) = 0 \) for some \( j \) in case of properties with weak collateral such as mobile homes. Here defaulting mortgage (if it exists) may be the only pareto-optimal solution in the absence of default-free mortgage (due to violation of the Basic condition of Proposition 1).
Note: For low levels of risk aversion, the above mortgage equilibria are *pareto-neutral*. This rationalizes the co-existence of bank and capital market financing.

**Figure 7: Mortgage Contracts Under Different States of Development of the Financial System and Their Pareto-Optimality: A Flowchart**
It is obvious that when both plain vanilla default-free and defaulting mortgage equilibria are pareto-neutral, they yield different property values (as they are derived endogenously from two different models as explicated in Propositions 1 and 2). However, when default-free mortgage strictly pareto-dominates its defaulting counterpart we observe a unique optimum property value.

(ii) \textit{Capital Market (Non-Bank) Equilibrium: A Pareto-enhancement Over Bank Based Equilibria.}

Since non-bank financiers like pension funds, insurance companies, etc., are allowed to own a fraction of property in the economy (along with making plain vanilla loans), $s'$ is not constrained to zero (as in the case of commercial banks in a specialized banking system). This (in general) moves the equilibria to a higher efficiency as it allows non-bank financiers to diversify cross-sectionally in property markets yielding $s' \geq 0$.\textsuperscript{20} Here, the interior solution (with strict inequality) resolves the real estate version of the \textit{asset location puzzle} as described by Geltner and Miller (2001). This result is attributed to adjustments of intertemporal marginal rate of substitution (IMRS) of both the financier and investor in such a way that no agent outbids the other for equity position in the property (see Equation 12a). However, allowing capital market based financiers like pension funds, insurance companies, etc. (with increased liquidity stemming from financial deepening) yields a decrease in interest rates and an increase in the price of property. This increase in the price of property can also be perceived as ensuing from an increase in its demand by the two competing agents in our economy.

(iii) \textit{Equilibrium Under Financial Innovation: A Further Pareto-Enhancement Over Non-Bank Financiers}

\textsuperscript{20} The trivial solution ($s' = 0$) for low risk averse agents yields an equilibrium, which is pareto-neutral to the default-free equilibrium in banks financing (see Ebrahim, 1996).
Since financial innovation allows capital market participants to invest in the quasi-equity instrument of Participating Mortgages, it makes their earlier portfolio (of default-free loan along with fractional purchase of property) redundant yielding a corner solution ($s' = 0$ – see Ebrahim, 1996).\footnote{Here too, the trivial solution ($\Phi = 0$) for low risk averse agents yields an equilibrium, which is pareto-neutral to the earlier equilibrium in non-banks financing with a portfolio of default-free plain vanilla mortgage and a fraction of property in the economy (see Ebrahim, 1996).} Default-free participating mortgage reduces agency costs beyond our second equilibrium as it:

(a) Ameliorates the underinvestment problem identified by Myers (1977) – This is because default-free participating mortgage not only collateralizes the tangible assets of a firm but also provides an indirect avenue (for financier) to participate in the property's payoffs. This sets the incentives (for investor) to maximize the property's market value in accordance with the prognosis of Bodie and Taggart (1978), and Schnabel, (1993) (as explicated below).

(b) Neutralizes the risk-shifting problem – While collateralization may resolve the underinvestment problem, there may remain an incentive for the property owner to engage in high risk (and possibly sub-optimal) property investments. Participating mortgage can solve the risk-shifting problem since the participation clause gives the financier some protection against property owner's incentive to choose properties with higher risk. Any such action by the property owner will increase the property price, but the financiers can prevent any expropriation of wealth by participating in property's payoffs. Hence, participating mortgages neutralizes the risk-shifting problem in accordance with the prognosis of Brennan and Schwartz (1982), Green, (1984) and Haugen and Senbet (1981, 1987).

A default-free participating mortgage is of advantage to capital-market-based financiers as it provides an indirect avenue for investing in property with a reduction in risk. This is due to the fact that the financier's downside risk applies only to a portion of its
return (i.e., the contingent-interest and/or shared-appreciation features of the mortgage loan) and not to the fixed-interest portion of the mortgage loan. This vehicle is also appealing to the investor (property owner) as reduction of risk exposure (of financier) is accompanied by a reduction in the overall cost of financing. A participating mortgage thus allows investor to retain full ownership, management, operation and control of the mortgaged property. This provides the incentive to efficiently manage and operate the mortgaged property to increase its net cash flow and value. This (in general) leads to an enhancement of welfare of both agents.

Default-free participating mortgage is allocatively efficient as the loan-to-value ratio is higher. The reason for this is that the loan includes the price of the option to share the profits of the property. However, the optimal participating loan parameters are a function of the risk preferences of the agents. This indicates a violation of the risk-neutral valuation principle of the Black and Scholes (1973) model, which states that the pricing of the derivatives securities is free of risk preferences. A key element of this argument is possibility of riskless hedge position consisting of long position in equity (common stock) and a short position in option. In case of property, arbitrage is generally not feasible for individual properties and therefore the results obtained are due to this unique feature.

III.c. Extension of the Model to a Universal Banking System

A universal banking system does not deter a bank from taking an equity position in the asset (property) market. Therefore, relaxing the constraint posed by Equation (10a) leads to the equilibrium akin to that of non-bank financiers like pension funds and insurance companies (in a specialized banking system). In other words, financing of property by banks in a universal banking system is more efficient than one in a specialized banking system corroborating the prognosis of Allen and Gale (2000), Holmstrom (1996) and Levine (2002). However, addition of financial deepening and financial innovation in capital markets leads to pareto-neutrality of
both specialized and universal banking systems. This confirms the assertions of the convergence of systems as reported by Dowd (1998) and Biswas and Lochel (2001).

IV. CONCLUDING REMARKS

The efficiency of financial system is of interest to investors, financiers and policy makers. Many developing countries have been moving towards a financial system skewed towards capital market financing in recent years without a clear consensus that such systems are necessarily better. Hence, it is important that growth and development theory addresses this debate to better inform policy makers as it significantly impacts on the country's economy.

This paper investigated the impact of financial development on property valuation in a rational expectations framework by modeling the agency theoretic perspective of risk averse investors (property owners) and financiers (banks/ capital markets). In contrast to previous research, we considered a setting in which financiers possess no inherent information processing or monitoring advantages. We demonstrated that property financing is undertaken in a pecking order of increasing pareto-efficiency (with reduction in the agency costs of debt) in a three staged process as financial architecture advances from a partially liberalized commercial bank to developed stage of capital markets. The primary solution is obtained in a rudimentary stage of commercial banks (in a specialized banking system), where the default-free mortgages are pareto-optimal to defaulting mortgages in accordance with the prognosis of Scott (1976) and Stulz and Johnson (1985). A pareto-improvement of the first solution is obtained by removing the constraint on asset ownership by universal banks, pension funds, insurance companies, etc. This solution resolves the real estate version of the asset location puzzle (see Geltner and Miller, 2001). A further pareto-enhancement of this equilibrium is obtained under financial innovation by embedding the above default-free mortgage with options (in the form of a participating mortgage) in accordance with the prognosis of Green (1984), Haugen and Senbet (1981, 1987) and Schnabel (1993).
Analysis of the above pareto-efficient financing packages revealed that the main link between the property owner and the lender is the intertemporal marginal rate of substitution (IMRS) of these two agents, which adjust in such a way to yield a unique equilibrium interest rate, loan amount and property value. Here our results discussed above differed from the well-known Capital-Structure Irrelevant Theorem (Modigliani and Miller, 1958). Our distinct interior solutions stem from the fact that, under a framework of risk-aversion (i.e., a non-linear valuation scheme), value additivity assumed in capital structure theorems does not hold as agents adjust their intertemporal marginal rate of substitution to own a property and to claim default-free mortgage claims against it with the lender (Varian, 1987). Further analysis also reveals that in general, defaulting mortgage equilibrium may not be feasible conforming to the prognosis of Myers (1977). However, when the defaulting mortgage equilibrium is feasible, it is at best pareto-neutral to default-free mortgage equilibrium. Thus valuation of properties under pareto-neutral (default-free and defaulting) mortgages constitutes a dilemma for an appraiser as they are contingent on equilibria.

Do these results have anything to say about how a financial system would evolve if left to its own mechanizations? Our results produce implications for financial system architecture and development. Our analysis predicts that an optimal financial system will configure itself skewed towards capital markets whether it originates from a specialized banking system or a universal banking system. A financial system in its infancy will be bank dominated, and increased financial development diminishes bank lending. Greater development in a financial system is manifested in several ways:

One is through financial liberalization, as in a liberalized financial system there are no restrictions on LTV ratio and financiers are not prohibited from direct ownership of property. Hence, capital markets develop as there are restrictions on banks activities. Hence we resolve the issue of how regulatory constraints, aimed principally at banking scope affect the evolution of financial system. Our results are in conformity with the empirical findings of Cho (1988), Chari & Henry (2003) and Demetriades et al (2001).
A second manifestation is through financial deepening (increase in the volume of credit being intermediated in financial market), our model proves that stock market liquidity enhances the supply of funds (which is a parameter of financial deepening) and subsequently the loan-to-value ratio of the mortgages, which results in enhancement of the allocative efficiency of capital and hence welfare improving (for more details, see Ebrahim and Mathur, 2000). Our results are in conformity with the empirical findings of King and Levine (1993) and Levine et al (2000).

A third manifestation is through financial innovation that further reduces endogenous agency costs of debt. In a well developed financial systems (capital market dominated), security design innovations provide richer set of risk management tools that permit greater customization of risk ameliorating instruments (Levine, 2002). Hence, our result differs from that of Modigliani and Miller (1958), Stiglitz (1974), Baron (1976) and Hellwig (1981) who argue that capital structure is irrelevant and that the form of securities issued is also unimportant, in other words, it disregards optimal security design and financial innovation. Thus, the welfare relevance of capital markets should grow through time as financial system develops. Our analysis is in conformity with that of Scharfstein (1988), Weinstein and Yafeh (1998) and Allen and Gale (2000). However, it differs with that of Rajan and Zingales (1999) and Tadesse (2001), who provide a strong case for bank financing.

We also rationalize the co-existence of banks and financial markets in a well-developed financial system. While in our model, the borrower either chooses bank financing or capital market financing, we could envision financing options lying along a continuum ranging from plain vanilla defaulting mortgage by banks to default-free participating mortgage by capital market financiers as the polar extremes. The key determining factors of financing choices are the quality of assets (a borrower can pose as collateral) and the risk profile of agents in the economy. The general results given above (of the pareto-advancement of equilibria) are valid under collateral that holds its value (in the future) and high risk aversion of agents. However, for
low quality collateral (such as mobile homes), where the terminal payoffs are zero for some state of the economy, defaulting mortgage financing (if feasible – under Proposition 2) may be the only option available as explicated in Footnote 21. Furthermore, agents with low risk aversion may be indifferent to bank and capital market financing as demonstrated in the trivial solutions explicated in Footnotes 22 and 23.

Although the measures of financial liberalization, financial deepening and financial innovation are not optimal, the results do provide a clear picture with sensible policy implications. Improving the functioning of the capital markets is critical for boosting long run economic growth. Thus, policy makers should focus on strengthening the overall efficiency of capital markets. However, as a future policy measure, it is important that the banks and capital markets are integrated. Finally, the policy-makers should ensure that this integration is orderly and properly sequenced (supported by, among other things, a sound financial sector and appropriate macroeconomic policies) in order to maximize the benefits from and minimize the risks associated with financial liberalization. In short, the bottom line is to strengthen the architecture of the financial system to lessen the frequency and severity of future disturbances.
REFERENCES


